Published by Institute of Physics Publishing for SISSA
Received: *June 15, 2005*
Accepted: *July 24, 2005*
Published: *August 16, 2005*

Supersymmetric ward identities and NMHV amplitudes involving gluinos

Steven J. Bidder, David C. Dunbar and Warren B. Perkins

Department of Physics, University of Wales Swansea
Swansea, SA2 8PP, U.K.
E-mail: 171370@swansea.ac.uk, d.c.dunbar@swan.ac.uk, w.perkins@swansea.ac.uk

Abstract: We show how Supersymmetric Ward Identities can be used to obtain amplitudes involving gluinos or adjoint scalars from purely gluonic amplitudes. We obtain results for all one-loop six-point NMHV amplitudes in $\mathcal{N} = 4$ Super Yang-Mills theory which involve two gluinos or two scalar particles. More general cases are also discussed.

Keywords: Extended Supersymmetry, NLO Computations.

Contents

1. Introduction

Recently, inspired by a possible duality between gauge theory and twistor string theory [1, 2], there has been much progress in obtaining one-loop gauge theory amplitudes in compact forms [3 − 10]. Most of the applications in loop calculations have been to amplitudes which involve only external gluons. In this paper we explore how amplitudes involving particles other than gluons may be obtained via symmetry constraints rather than by direct computation. In particular, we use Supersymmetric Ward Identities [11](SWI) to obtain amplitudes involving gluinos and scalars from purely gluonic amplitudes.

On-shell SWI [11] impose powerful constraints on amplitudes in gauge theories, giving algebraic constraints between amplitudes with the same helicity configuration but different external particle types. These constraints apply at any order in perturbation theory. From a Feynman diagram perspective, these relationships are most naturally employed to obtain purely gluonic amplitudes from amplitudes involving fermions, as the latter are easier to calculate. For example, at six-point, Kunszt [12] was able to obtain the purely gluonic tree amplitudes from the set of amplitudes with four gluons and two fermions. Motivated by the recent advances in calculating purely gluonic amplitudes, in this paper we will reverse this process and generate amplitudes involving fermions from the purely gluonic ones. For some helicity configurations the SWI contain sufficient information to simply solve for the fermionic amplitudes: for example in $\mathcal{N} = 4$ gauge theory the SWI for amplitudes with two negative helicities and the rest positive (known as MHV amplitudes) can be easily solved and amplitudes with any external particles obtained from the purely gluonic MHV amplitudes [13, 14] by a simple multiplicative factor.

For other configurations, such as those with three negative helicities (known as "next-to-MHV" or NMHV amplitudes) the SWI do not allow such simple solutions. However, we shall show how the SWI can be solved in a natural way to obtain amplitudes with two gluinos in terms of the purely gluonic case. We will first apply this to the six-point tree amplitudes where we can connect to known expressions. Secondly we shall determine the one-loop six-point NMHV amplitudes in $\mathcal{N} = 4$ SYM which involve two gluinos. More generally there also exist SWI which involve amplitudes with two gluinos, four gluinos, two scalars and two gluinos plus a scalar. We explicitly determine the two scalar amplitudes. The SWI then give the remaining amplitudes directly in terms of known amplitudes.

2. $\mathcal{N} = 4$ amplitudes and recent developments

In this section, we describe the organisation of tree and one-loop amplitudes and review the recent progress in determining the one-loop gluonic amplitudes in $\mathcal{N} = 4$ SYM.

Tree amplitudes: tree-level amplitudes for $\mathrm{U}(N_c)$ or $\mathrm{SU}(N_c)$ gauge theories with n external adjoint particles can be decomposed into colour-ordered partial amplitudes multiplied by an associated colour-trace [15, 16]. Summing over all non-cyclic permutations reconstructs the full amplitude $\mathcal{A}_n^{\rm tree}$ from the partial amplitudes $A_n^{\rm tree}(\sigma)$,

$$\mathcal{A}_n^{\rm tree}(\{k_i, a_i\}) = g^{n-2} \sum_{\sigma \in S_n/Z_n} \mathrm{Tr}(T^{a_{\sigma(1)}} \cdots T^{a_{\sigma(n)}})\, A_n^{\rm tree}(k_{\sigma(1)}, \ldots, k_{\sigma(n)})\,, \tag{2.1}$$

where k_i and a_i are respectively the momentum and colour-index of the i-th external particle, g is the coupling constant and S_n/Z_n is the set of non-cyclic permutations of $\{1, \ldots, n\}$. The $\mathrm{U}(N_c)$ ($\mathrm{SU}(N_c)$) generators T^a are the set of traceless hermitian $N_c \times N_c$ matrices, normalised such that $\mathrm{Tr}\left(T^a T^b\right) = \delta^{ab}$. Conventionally we take all particles to be outgoing. We denote gluons by g_i and adjoint fermions by Λ_i. We will often refer to the adjoint fermions as gluinos for simplicity.

Amplitudes involving fundamental particles, for example fermions (or quarks) λ_i have a different decomposition [15],

$$\mathcal{A}_n^{\rm tree}(\bar\lambda_1, \lambda_2, g_3, \ldots,) = g^{n-2} \sum_{\sigma \in S_{n-2}} (T^{a_{\sigma(3)}} \cdots T^{a_{\sigma(n)}})_{i_2}{}^{\bar{\imath}_1}\; A_n^{\rm tree}(\bar\lambda_1, \lambda_2, g_{\sigma(3)}, \ldots, g_{\sigma(n)})\,, \tag{2.2}$$

where i_2 and $\bar{\imath}_1$ are the colour indices on the quarks. Note that the two quarks are adjacent in the ordering. The partial tree amplitudes for two quarks are simply related to those of adjoint fermions by,

$$A_n^{\rm tree}(\bar\lambda_1^+, \lambda_2^-, g_3, g_4 \cdots g_n) = A_n^{\rm tree}(\bar\Lambda_1^+, \Lambda_2^-, g_3, g_4 \cdots g_n)\,, \tag{2.3}$$

and the difference between the full amplitudes is entirely in the colour factors. For amplitudes where exactly two of the external particles are quarks or gluinos and the remainder are gluons, the amplitude will vanish unless the quarks or gluinos have opposite helicity.

Tree amplitudes where all the particles have the same helicity vanish, as do amplitudes where all but one of the helicities are identical,

$$A_n^{\rm tree}(1^\pm, 2^+, \ldots, n^+) = 0\,. \tag{2.4}$$

The simplest non-vanishing amplitudes are the MHV amplitudes with two particles of negative helicity and the remainder positive. The MHV partial amplitudes for gluons are given by the Parke-Taylor formulae [13],

$$A_n^{\rm tree}(g_1^+, \ldots, g_j^-, \ldots, g_k^-, \ldots, g_n^+) = i\,\frac{\langle j\,k\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle \cdots \langle n\,1\rangle}\,, \tag{2.5}$$

for a partial amplitude where j and k are the legs with negative helicity. We use the notation $\langle j\,l\rangle \equiv \langle j^-|l^+\rangle$, $[j\,l] \equiv \langle j^+|l^-\rangle$, with $|i^\pm\rangle$ being a massless Weyl spinor with momentum k_i and chirality $\pm$ [17, 16]. The spinor products are related to momentum invariants by $\langle i\,j\rangle\,[j\,i] = 2k_i\cdot k_j \equiv s_{ij}$ with $\langle i\,j\rangle^* = [j\,i]$.

The MHV amplitudes with external particles other than gluons can be obtained from these using SWI. Formulae linking these amplitudes can also be derived using current algebra techniques [14]. Formulae for amplitudes with three minus helicity gluons can be deduced by recursion relations but are more complicated [18].

One-loop amplitudes: for one-loop amplitudes of adjoint particles, one may perform a colour decomposition similar to the tree-level decomposition (2.1) [19]. In this case there are two traces over colour matrices and the result takes the form,

$$\mathcal{A}_n^{1-\rm loop}\left(\{k_i, a_i\}\right) = g^n \sum_{c=1}^{\lfloor n/2\rfloor+1} \sum_{\sigma\in S_n/S_{n;c}} {\rm Gr}_{n;c}\left(\sigma\right)\, A_{n;c}(\sigma)\,, \tag{2.6}$$

where $\lfloor x\rfloor$ is the largest integer less than or equal to x. The leading colour-structure factor,

$${\rm Gr}_{n;1}(1) = N_c\ \ {\rm Tr}\left(T^{a_1}\cdots T^{a_n}\right)\,, \tag{2.7}$$

is just N_c times the tree colour factor, and the subleading colour structures ($c > 1$) are given by,

$${\rm Gr}_{n;c}(1) = {\rm Tr}\left(T^{a_1}\cdots T^{a_{c-1}}\right)\,{\rm Tr}\left(T^{a_c}\cdots T^{a_n}\right)\,. \tag{2.8}$$

S_n is the set of all permutations of n objects and $S_{n;c}$ is the subset leaving ${\rm Gr}_{n;c}$ invariant. Once again it is convenient to use $\mathrm{U}(N_c)$ matrices; the extra $\mathrm{U}(1)$ decouples [19].

For one-loop amplitudes the subleading in colour amplitudes $A_{n;c}$, $c > 1$, may be obtained from summations of permutations of the leading in colour amplitude [20],

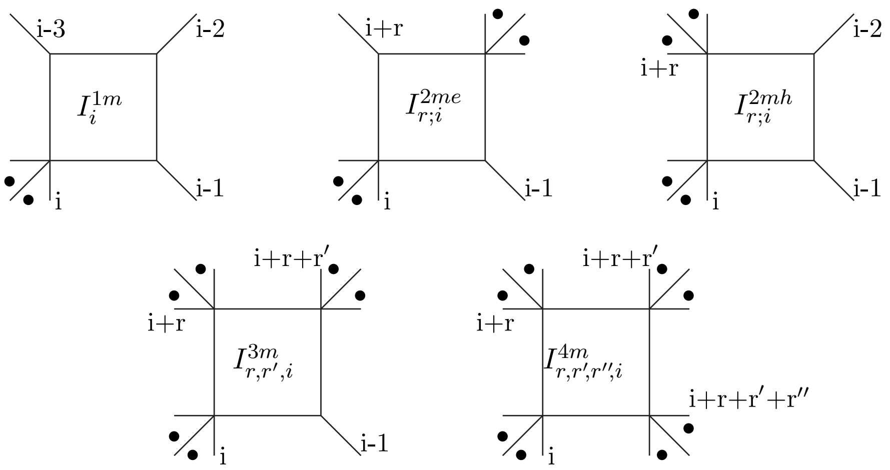
$$A_{n;c}(1, 2, \ldots, c-1; c, c+1, \ldots, n) = (-1)^{c-1} \sum_{\sigma\in COP\{\alpha\}\{\beta\}} A_{n;1}(\sigma)\,, \tag{2.9}$$

where $\alpha_i \in \{\alpha\} \equiv \{c-1, c-2, \ldots, 2, 1\}$, $\beta_i \in \{\beta\} \equiv \{c, c+1, \ldots, n-1, n\}$, and $COP\{\alpha\}\{\beta\}$ is the set of all permutations of $\{1, 2, \ldots, n\}$ with n held fixed that preserve the cyclic ordering of the α_i within $\{\alpha\}$ and of the β_i within $\{\beta\}$, while allowing for all possible relative orderings of the α_i with respect to the β_i. Hence, we need only focus on the leading in colour amplitude $A_{n;1}$ (which we will generally abbreviate to A_n) and use this relationship to generate the full amplitude if required.

One-loop amplitudes depend on the particles circulating within the loop and thus on the spectrum of the theory. In supersymmetric amplitudes there are generically cancellations between the bosons and fermions in the loop. For $\mathcal{N} = 4$ SYM these cancellations lead to considerable simplifications in the loop momentum integrals. This is manifest in the "string-based approach" to computing loop amplitudes [21]. As a result of these simplifications, $\mathcal{N} = 4$ one-loop amplitudes can be expressed simply as a sum of scalar box-integral functions [20],

$$I_i^{\rm 1m} \quad I_{r;i}^{\rm 2m\mathit{e}} \quad I_{r;i}^{\rm 2m\mathit{h}} \quad I_{r,r',i}^{\rm 3m} \quad I_{r,r',r'',i}^{\rm 4m} \tag{2.10}$$

with the labeling as indicated,



Explicit forms for these scalar box integrals can be found in ref. [20]. The four dimensional boxes have dimension -2. It is convenient to define dimension zero F-functions by removing the momentum prefactors of the $D = 4$ scalar boxes [22],

$$I_i^{\rm 1m} = -2c_\Gamma \frac{F_i^{\rm 1m}}{t_{i-3}^{[2]} t_{i-2}^{[2]}}\,, \quad I_{r;i}^{\rm 2m\mathit{e}} = -2c_\Gamma \frac{F_{r;i}^{\rm 2m\,\mathit{e}}}{t_{i-1}^{[r+1]} t_i^{[r+1]} - t_i^{[r]} t_{i+r+1}^{[n-r-2]}}\,, \quad I_{r;i}^{\rm 2m\mathit{h}} = -2c_\Gamma \frac{F_{r;i}^{\rm 2m\,\mathit{h}}}{t_{i-2}^{[2]} t_{i-1}^{[r+1]}}\,,$$
$$I_{r,r',i}^{\rm 3m} = -2c_\Gamma \frac{F_{r,r';i}^{\rm 3m}}{t_{i-1}^{[r+1]} t_i^{[r+r']} - t_i^{[r]} t_{i+r+r'}^{[n-r-r'-1]}}\,,\,, \quad I_{r,r',r'',i}^{\rm 4m} = -2 \frac{F_{r,r',r'';i}^{\rm 4m}}{t_i^{[r+r']}\, t_{i+r}^{[r'+r'']}\, \rho}\,, \tag{2.11}$$

where,

$$t_a^{[p]} \equiv (k_a + k_{a+1} + \cdots + k_{a+p-1})^2\,, \tag{2.12}$$

and,

$$c_\Gamma = \frac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{(4\pi)^{2-\epsilon}\Gamma(1-2\epsilon)}\,. \tag{2.13}$$

The one-loop amplitudes can thus be expressed as,

$$A^{\mathcal{N}=4} = \sum_i c_i F_i \,, \tag{2.14}$$

and the computation of one-loop $\mathcal{N} = 4$ amplitudes is then just a matter of determining the rational coefficients c_i. These remarkable simplifications also appear to extend beyond one-loop [23].

Furthermore, it has been shown that these amplitudes are "cut-constructible", in that the coefficients can be determined from unitary cuts. Using this fact, in ref [20] the one-loop amplitudes were determined for the all-n MHV amplitudes and in [22] the remaining six-point gluonic amplitudes (the NMHV amplitudes) were computed and the MHV amplitudes determined in $\mathcal{N} = 1$ theories.

The amplitude for gluonic scattering in $\mathcal{N} = 4$ theory can be thought of as a component of gluonic scattering in non-supersymmetric theories. One can decompose the one-loop pure gluon amplitude as a sum of contributions from matter supermultiplets,

$$A_n \;\equiv\; A_n^{\mathcal{N}=4} - 4A_n^{\mathcal{N}=1 \text{ chiral}} + A_n^{[0]} \,, \tag{2.15}$$

where $A_n^{[0]}$ is the contribution from the complex scalar (or $\mathcal{N} = 0$ matter multiplet) circulating in the loop. (Throughout we assume the use of a supersymmetry preserving regulator [24, 21, 25].)

Recent progress: recently there has been a great deal of progress in calculating perturbative amplitudes in compact forms: this is key to our obtaining gluino amplitudes. Many of the techniques can be applied directly to amplitudes involving particles other than gluons however our philosophy will be to avoid such direct computations but rather to exploit the gluonic amplitudes via the SWI. We have verified on occasion our results using these methods — which we now review.

Progress in calculating amplitudes has been remarkable and varied. At tree level, inspired by the duality between topological string theory and gauge theory [1], a reformulation of perturbation theory in terms of MHV-vertices was proposed [2]. This promoted the MHV amplitudes of eq. (2.5) to the role of fundamental building blocks in the perturbative expansion. By continuing legs off-shell in a well specified manner these could be sewn together to form other amplitudes. This reformulation, although still lacking a field theory proof, produces relatively compact expressions for tree amplitudes. Although initially presented for purely gluonic amplitudes, it has been successfully extended to other particle types [26, 27].

In a different development, a series of recursion relations for calculating tree amplitudes have been postulated [28]. These yield compact expressions for gluonic tree amplitudes [29], the six-point NMHV amplitudes involving fermions [30] and gravity amplitudes [31 – 33].

Although impressive, progress in calculating tree amplitudes has generally involved producing better forms for amplitudes which were previously available (if only numerically). A much tougher but more rewarding problem is to compute loop amplitudes for which much less is known. For one-loop amplitudes in gauge theory, full results for all helicities and all particle types are only known for the four-point [34, 25] and five-point [35 – 38] amplitudes. Beyond five-point, the one-loop amplitudes are much better understood within supersymmetric theories.

The MHV vertex approach has been shown to extend to one-loop in ref. [39], where the one-loop $\mathcal{N} = 4$ MHV amplitudes were computed and shown to be in complete agreement with the results of [20], and in refs [40, 41] where the $\mathcal{N} = 1$ MHV one-loop amplitudes were computed and shown to be in agreement with the results of [22]. Although the MHV vertex approach appears to work in principle, the connection to the form of eq. (2.14) involves integration. Progress in evaluating the c_i has been more fruitful when employing methods which determine the coefficients using algebraic equations. Techniques based on the structure of the amplitude in twistor space can be used to give algebraic equations for the box coefficients [6, 42, 7, 8] and techniques based on unitarity [20, 22] can evaluate the coefficients by evaluating the cuts of the amplitude. For example, the box-coefficients must satisfy a coplanarity condition in twistor space,

$$K_{ijkl} c^{NMHV} = 0 \,, \tag{2.16}$$

where,

$$K_{ijkl} = \langle i\, j \rangle\, \epsilon^{\dot{a}\dot{b}} \frac{\partial}{\partial \tilde{\lambda}_k^{\dot{a}}} \frac{\partial}{\partial \tilde{\lambda}_l^{\dot{b}}} + \text{perms} \,, \tag{2.17}$$

when the amplitude is expressed as a function of spinor variables $k_{a\dot{a}} = \lambda_a \tilde{\lambda}_a$. The coplanarity of the box-coefficients for the $\mathcal{N} = 4$ amplitudes was shown in refs. [3, 43] and shown to extend to $\mathcal{N} < 4$ theories in in [44, 9]. The box-coefficients we compute for amplitudes involving gluinos also satisfy eq. (2.16).

These techniques have been very successful and results include the recent computation of all $\mathcal{N} = 4$ NMHV one-loop amplitudes [3, 4] and various next-to-next-to-MHV (N^2MHV) box coefficients [5]. An important development, which enhances the power of the unitarity method, is the observation by Britto, Cachazo and Feng [5] that box integral coefficients can be obtained from generalised unitarity cuts [45, 46, 3] by analytically continuing the massless corners of the quadruple cuts. The quadruple cuts give the box-coefficients as a product of four tree amplitudes,

$$\begin{aligned} c = \frac{1}{2} \sum_{\mathcal{S}} \Big(& A^{\text{tree}}(\ell_1, i_1, \ldots, i_2, \ell_2) \times A^{\text{tree}}(\ell_2, i_3, \ldots, i_4, \ell_3) \\ & \times A^{\text{tree}}(\ell_3, i_5, \ldots, i_6, \ell_4) \times A^{\text{tree}}(\ell_4, i_7, \ldots, i_8, \ell_1) \Big) \,. \end{aligned} \tag{2.18}$$

The sum is over all allowed intermediate configurations and particle types [5] where the cut legs are frozen in a specific manner. This formula could be used to compute the amplitudes involving gluinos, however using the SWI produces compact formulae in a straightforward manner. These formulae can be numerically compared to the forms produced from (2.18) as consistency checks.

These techniques are also useful in calculating amplitudes in $\mathcal{N} < 4$ theories [8, 44, 9, 10], although these amplitudes are more complicated and contain integral functions

JHEP08(2005)055

other than the box functions. Unfortunately, non-supersymmetric theories are not cut-constructible [22], so the unitary techniques are not immediately applicable, although progress is ongoing in this area [47, 48].

3. Supersymmetric ward identities

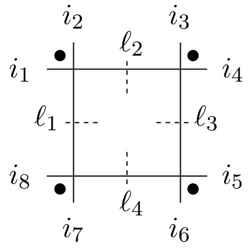


Figure 1: A quadruple cut of a n-point amplitude. The dashed lines represent the cuts. The dots represent arbitrary numbers of external line insertions.

Supersymmetric Ward Identities relate amplitudes with the same helicity structure but with different external particles types. The Ward identities can be obtained by acting with the supersymmetry generator Q on a string of operators, z_i, which has vanishing vacuum expectation value. Typical choices are strings with an odd number of fermionic operators. Since Q annihilates the vacuum we obtain,

$$0 = \left\langle \left[Q, \prod_i z_i \right] \right\rangle = \sum_i \left\langle z_1 \cdots [Q, z_i] \cdots z_n \right\rangle . \tag{3.1}$$

For $\mathcal{N} = 1$ supersymmetry we can use the supersymmetry algebra,

$$\begin{aligned} [Q(\eta), g^+(p)] &= -\Gamma^+(p,\eta)\bar{\Lambda}^+ , \qquad [Q(\eta), g^-(p)] = \Gamma^-(p,\eta)\Lambda^- , \\ [Q(\eta), \bar{\Lambda}^+(p)] &= -\Gamma^-(p,\eta) g^+ , \qquad [Q(\eta), \Lambda^-(p)] = \Gamma^+(p,\eta) g^- , \end{aligned} \tag{3.2}$$

where $g^\pm(p)$ is the operator creating a gluon of momentum p and the supersymmetry generator $Q(\eta)$ depends on a spinor parameter η. The $\Gamma^\pm$ are,

$$\Gamma^+(p,\eta) \equiv [\eta\, p] , \qquad \Gamma^-(p,\eta) \equiv \langle p\, \eta \rangle . \tag{3.3}$$

Applying this to $A'_n(g_1^-, g_2^-, \Lambda_3^+, g_4^+, \ldots, g_n^+)$ (i.e. a string of glue creation operators with a single gluino creation operator) we obtain,

$$\begin{aligned} 0 = &\ \langle 1\, \eta \rangle\, A_n(\Lambda_1^-, g_2^-, \bar{\Lambda}_3^+, g_4^+, \ldots, g_n^+) + \langle 2\, \eta \rangle\, A_n(g_1^-, \Lambda_2^-, \bar{\Lambda}_3^+, g_4^+, \ldots, g_n^+) - \\ &- \langle 3\, \eta \rangle\, A_n(g_1^-, g_2^-, g_3^+, g_4^+, \ldots, g_n^+) , \end{aligned} \tag{3.4}$$

where we have used the fact that amplitudes with two fermions of the same helicity vanish. Choosing $\eta = 1$, for example, gives,

$$A_n(g_1^-, \Lambda_2^-, \bar{\Lambda}_3^+, g_4^+, \ldots, g_n^+) = \frac{\langle 3\, 1 \rangle}{\langle 2\, 1 \rangle} A_n(g_1^-, g_2^-, g_3^+, g_4^+, \ldots, g_n^+) , \tag{3.5}$$

and we can thus obtain the MHV two-gluino amplitudes from the gluonic amplitude.

For NMHV amplitudes the SWI do not lead to such simple solutions: applying the supersymmetry operator to $A_n(g_1^-, g_2^-, g_3^-, \bar{\Lambda}_4^+, g_5^+, \ldots, g_n^+)$ we obtain,

$$\begin{aligned} 0 = &\ \langle 1\, \eta \rangle\, A_n(\Lambda_1^-, g_2^-, g_3^-, \bar{\Lambda}_4^+, g_5^+, \ldots, g_n^+) + \langle 2\, \eta \rangle\, A_n(g_1^-, \Lambda_2^-, g_3^-, \bar{\Lambda}_4^+, g_5^+, \ldots, g_n^+) + \\ &+ \langle 3\, \eta \rangle\, A_n(g_1^-, g_2^-, \Lambda_3^-, \bar{\Lambda}_4^+, g_5^+, \ldots, g_n^+) - \langle 4\, \eta \rangle\, A_n(g_1^-, g_2^-, g_3^-, g_4^+, g_5^+, \ldots, g_n^+) . \end{aligned} \tag{3.6}$$

This system has rank 2, so it can only directly give two of the amplitudes in terms of the other two. This relationship was used originally [12] to obtain the six point gluonic amplitude from the two-fermion amplitudes. By itself, this relationship does not allow us to solve for the fermionic amplitudes unambiguously from the purely gluonic. However, when we apply further constraints we will be able to obtain the fermionic amplitudes.

We can also consider $\mathcal{N}=2$ Supersymmetric Ward Identities [11, 49]. Using supersymmetry generators Q_a, $a=1,2$, we have,

$$\begin{aligned}
[Q_a(\eta), g^+(p)] &= -\Gamma^+(p,\eta)\bar\Lambda_a^+\,, \qquad [Q_a(\eta), g^-(p)] = \Gamma^-(p,\eta)\Lambda_a^-\,,\\
[Q_a(\eta), \bar\Lambda_b^+(p)] &= -\Gamma^-(p,\eta)\delta_{ab}g^+ - i\Gamma^+(p,\eta)\epsilon_{ab}\phi^+\,,\\
[Q_a(\eta), \Lambda_b^-(p)] &= \Gamma^+(p,\eta)\delta_{ab}g^- + i\Gamma^-(p,\eta)\epsilon_{ab}\phi^-\,,\\
[Q_a(\eta), \phi^+(p)] &= -i\Gamma^-(p,\eta)\epsilon_{ab}\bar\Lambda_b^+\,, \qquad [Q_a(\eta), \phi^-(p)] = +i\Gamma^+(p,\eta)\epsilon_{ab}\Lambda_b^-\,.
\end{aligned} \tag{3.7}$$

We will need to use these identities to determine amplitudes involving scalars or two flavours of gluino.

4. Tree amplitudes

In this section we demonstrate how to generate tree amplitudes involving two gluinos from purely gluonic tree amplitudes and then compare these to the known expressions obtained via recursion relations [30, 10] which themselves agree with the Feynman diagram computations [12].

Six-point NMHV tree amplitudes. MHV amplitudes with two gluinos have been discussed in section 2, in this section we consider NMHV amplitudes and compare our results with previous calculations [12, 30]. For colour-ordered gluonic tree amplitudes there are three independent NMHV helicity configurations. When we consider amplitudes with two fermions and four gluons there are considerably more depending on the position of the two fermions. This set can be reduced considerably using the U(1) decoupling (or "dual Ward") identity [16]. However, we will not explicitly use these identities since they do not extend to one-loop level, or more precisely, they have different implications. We shall in this section restrict ourselves to adjoint fermions (gluinos).

We first consider amplitudes derived from the gluonic amplitude,

$$\begin{aligned}
A_6^{\rm tr}(g_1^-, g_2^-, g_3^-, g_4^+, g_5^+, g_6^+) = {}& \frac{i\langle 4|K_{234}|1\rangle^3}{t_{234}\,[2\,3]\,[3\,4]\,\langle 5\,6\rangle\,\langle 6\,1\rangle\,\langle 2|K_{234}|5\rangle} + \\
&+ \frac{i\langle 6|K_{345}|3\rangle^3}{t_{612}\,[6\,1]\,[1\,2]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 2|K_{345}|5\rangle}\,,
\end{aligned} \tag{4.1}$$

where, $\langle A|K_{abc}|B\rangle \equiv \langle A^+|\not{k}_a + \not{k}_b + \not{k}_c|B^+\rangle = [A\,a]\,\langle a\,B\rangle + [A\,b]\,\langle b\,B\rangle + [A\,c]\,\langle c\,B\rangle$. The amplitudes involving two fermions which are related to this purely gluonic amplitude can be obtained by conjugation, relabeling and *flipping* (i.e. using $A(1234556) = A(654321)$) from the following four,

$$\begin{aligned}
&A_6^{\rm tr}(\Lambda_1^-, g_2^-, g_3^-, \bar\Lambda_4^+, g_5^+, g_6^+)\,, \qquad A_6^{\rm tr}(g_1^-, \Lambda_2^-, g_3^-, \bar\Lambda_4^+, g_5^+, g_6^+)\,,\\
&A_6^{\rm tr}(g_1^-, g_2^-, \Lambda_3^-, \bar\Lambda_4^+, g_5^+, g_6^+)\,, \qquad A_6^{\rm tr}(g_1^-, \Lambda_2^-, g_3^-, g_4^+, \bar\Lambda_5^+, g_6^+)\,.
\end{aligned} \tag{4.2}$$

The SWI (3.6) relating the first three of these amplitudes to the gluonic amplitude has rank 2 and hence, in principle, is not sufficient to determine the fermionic amplitudes in terms of the gluonic. However, when we utilise their inherent symmetries, we can unambiguously determine these fermionic amplitudes. The basic idea is to look for identities of the form,

$$A\,\langle 1\,\eta\rangle + B\,\langle 2\,\eta\rangle + C\,\langle 3\,\eta\rangle - D\,\langle 4\,\eta\rangle = 0\,, \tag{4.3}$$

where the form of D is motived by the terms in the numerator of the compact expressions for the gluonic tree amplitudes. We shall search for solutions where A, B and C are *polynomial* in the spinor invariants $\langle i\,j\rangle$ and $[i\,j]$, so that the gluino amplitudes are free from spurious singularities and poles.

Equation (4.1) contains two terms which we examine individually. Writing the second term as $\langle 6|K_{612}|3\rangle X$ and focusing on the the $\langle 6|K_{612}|3\rangle$ factor, the Schouten identity yields,

$$\begin{aligned}\langle 6|K_{612}|3\rangle\,\langle 4\,\eta\rangle &= -\langle 6|K_{612}|\eta\rangle\,\langle 3\,4\rangle + \langle 6|K_{612}|4\rangle\,\langle 3\,\eta\rangle\\ &= \langle 6|K_{612}|4\rangle\,\langle 3\,\eta\rangle - [6\,1]\,\langle 3\,4\rangle\,\langle 1\,\eta\rangle - [6\,2]\,\langle 3\,4\rangle\,\langle 2\,\eta\rangle\ .\end{aligned} \tag{4.4}$$

This implies that the following are solutions of the SWI,

$$\begin{aligned}A_6^{\rm tr}(\Lambda_1^-, g_2^-, g_3^-, \bar\Lambda_4^+, g_5^+, g_6^+) &= -\,[6\,1]\,\langle 3\,4\rangle\,X\\ A_6^{\rm tr}(g_1^-, \Lambda_2^-, g_3^-, \bar\Lambda_4^+, g_5^+, g_6^+) &= -\,[6\,2]\,\langle 3\,4\rangle\,X\\ A_6^{\rm tr}(g_1^-, g_2^-, \Lambda_3^-, \bar\Lambda_4^+, g_5^+, g_6^+) &= \langle 6|K_{612}|4\rangle X\\ A_6^{\rm tr}(g_1^-, g_2^-, g_3^-, g_4^+, g_5^+, g_6^+) &= \langle 6|K_{612}|3\rangle X\ .\end{aligned} \tag{4.5}$$

Similarly, writing the first term as $\langle 4|K_{234}|1\rangle Y$ we find,

$$\langle 4|K_{234}|1\rangle\,\langle 4\,\eta\rangle = \langle 1|K_{234}4|\eta\rangle = t_{234}\,\langle 1\,\eta\rangle - \langle 2|K_{234}|1\rangle\,\langle 2\,\eta\rangle - \langle 3|K_{234}|1\rangle\,\langle 3\,\eta\rangle\ , \tag{4.6}$$

which suggests a second solution to the SWI of the form,

$$\begin{aligned}A_6^{\rm tr}(\Lambda_1^-, g_2^-, g_3^-, \bar\Lambda_4^+, g_5^+, g_6^+) &= t_{234}Y\\ A_6^{\rm tr}(g_1^-, \Lambda_2^-, g_3^-, \bar\Lambda_4^+, g_5^+, g_6^+) &= -\langle 2|K_{234}|1\rangle Y\\ A_6^{\rm tr}(g_1^-, g_2^-, \Lambda_3^-, \bar\Lambda_4^+, g_5^+, g_6^+) &= -\langle 3|K_{234}|1\rangle Y\\ A_6^{\rm tr}(g_1^-, g_2^-, g_3^-, g_4^+, g_5^+, g_6^+) &= \langle 4|K_{234}|1\rangle Y\ .\end{aligned} \tag{4.7}$$

The two gluino tree amplitudes are thus,

$$\begin{aligned}A_6^{\rm tr}(g_1^-, g_2^-, \Lambda_3^-, \bar\Lambda_4^+, g_5^+, g_6^+) &= -\frac{i\langle 4|K_{234}|1\rangle^2\langle 3|K_{234}|1\rangle}{t_{234}\,[2\,3]\,[3\,4]\,\langle 5\,6\rangle\,\langle 6\,1\rangle\,\langle 2|K_{234}|5\rangle} +\\ &\quad + \frac{i\langle 6|K_{612}|3\rangle^2\langle 6|K_{612}|4\rangle}{t_{612}\,[6\,1]\,[1\,2]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 2|K_{612}|5\rangle}\\ A_6^{\rm tr}(g_1^-, \Lambda_2^-, g_3^-, \bar\Lambda_4^+, g_5^+, g_6^+) &= -\frac{i\langle 4|K_{234}|1\rangle^2\langle 2|K_{234}|1\rangle}{t_{234}\,[2\,3]\,[3\,4]\,\langle 5\,6\rangle\,\langle 6\,1\rangle\,\langle 2|K_{234}|5\rangle} +\\ &\quad + \frac{i\langle 6|K_{612}|3\rangle^2\,[2\,6]\,\langle 3\,4\rangle}{t_{612}\,[6\,1]\,[1\,2]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 2|K_{612}|5\rangle}\end{aligned}$$

$$A_6^{\rm tr}(\Lambda_1^-, g_2^-, g_3^-, \bar\Lambda_4^+, g_5^+, g_6^+) = \frac{i\langle 4|K_{234}|1\rangle^2 t_{234}}{t_{234}\,[2\,3]\,[3\,4]\,\langle 5\,6\rangle\,\langle 6\,1\rangle\,\langle 2|K_{234}|5\rangle} + \\ + \frac{i\langle 6|K_{612}|3\rangle^2\,[1\,6]\,\langle 3\,4\rangle}{t_{612}\,[6\,1]\,[1\,2]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 2|K_{612}|5\rangle}\,. \tag{4.8}$$

In principle there is some ambiguity in these solutions since the coefficients of $\langle 6|K_{612}|3\rangle$ and $\langle 4|K_{234}|1\rangle$ are not unique:

$$\langle 6|K_{612}|3\rangle X + \langle 4|K_{234}|1\rangle Y = \langle 6|K_{612}|3\rangle\left(X + \frac{Z}{\langle 6|K_{612}|3\rangle}\right) + \langle 4|K_{234}|1\rangle\left(Y - \frac{Z}{\langle 4|K_{234}|1\rangle}\right). \tag{4.9}$$

However, by taking X and Y to be the values that appear in the gluon amplitudes we do not introduce any of the unphysical singularities/poles that arise in the general ($Z \neq 0$) case. The remaining amplitude, $A_6^{\rm tr}(g_1^-, \Lambda_2^-, g_3^-, g_4^+, \bar\Lambda_5^+, g_6^+)$, can be obtained from the SWI,

$$0 = \langle 1\,\eta\rangle\, A_6^{\rm tr}(\Lambda_1^-, g_2^-, g_3^-, g_4^+, \bar\Lambda_5^+, g_6^+) + \langle 2\,\eta\rangle\, A_6^{\rm tr}(g_1^-, \Lambda_2^-, g_3^-, g_4^+, \bar\Lambda_5^+, g_6^+) + \\ + \langle 3\,\eta\rangle\, A_6^{\rm tr}(g_1^-, g_2^-, \Lambda_3^-, g_4^+, \bar\Lambda_5^+, g_6^+) - \langle 5\,\eta\rangle\, A_6^{\rm tr}(g_1^-, g_2^-, g_3^-, g_4^+, g_5^+, g_6^+)\,, \tag{4.10}$$

which is obtained by acting with Q on $A_6^{\rm tr}(g_1^-, g_2^-, g_3^-, g_4^+, \bar\Lambda_5^+, g_6^+)$. Here we use the identities,

$$\langle 6|K_{612}|3\rangle\,\langle 5\,\eta\rangle = \langle 6|K_{612}|5\rangle\,\langle 3\,\eta\rangle - [6\,1]\,\langle 3\,5\rangle\,\langle 1\,\eta\rangle - [6\,2]\,\langle 3\,5\rangle\,\langle 2\,\eta\rangle\,, \\ \langle 4|K_{234}|1\rangle\,\langle 5\,\eta\rangle = \langle 4|K_{234}|5\rangle\,\langle 1\,\eta\rangle - [4\,2]\,\langle 1\,5\rangle\,\langle 2\,\eta\rangle - [4\,3]\,\langle 1\,5\rangle\,\langle 3\,\eta\rangle\,, \tag{4.11}$$

to obtain,

$$A_6^{\rm tr}(g_1^-, \Lambda_2^-, g_3^-, g_4^+, \bar\Lambda_5^+, g_6^+) = \frac{-i\langle 4|K_{234}|1\rangle^2\,[4\,2]\,\langle 1\,5\rangle}{t_{234}\,[2\,3]\,[3\,4]\,\langle 5\,6\rangle\,\langle 6\,1\rangle\,\langle 2|K_{234}|5\rangle} - \\ - \frac{i\langle 6|K_{612}|3\rangle^2\,[6\,2]\,\langle 3\,5\rangle}{t_{612}\,[6\,1]\,[1\,2]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 2|K_{612}|5\rangle}\,. \tag{4.12}$$

This SWI also yields consistent but independent expressions for two of the amplitudes found previously. For example,

$$A_6^{\rm tr}(\Lambda_1^-, g_2^-, g_3^-, g_4^+, \bar\Lambda_5^+, g_6^+) = \frac{i\langle 4|K_{234}|1\rangle^2\langle 4|K_{234}|5\rangle}{t_{234}\,[2\,3]\,[3\,4]\,\langle 5\,6\rangle\,\langle 6\,1\rangle\,\langle 2|K_{234}|5\rangle} - \\ - \frac{i\langle 6|K_{612}|3\rangle^2\,[6\,1]\,\langle 3\,5\rangle}{t_{612}\,[6\,1]\,[1\,2]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 2|K_{612}|5\rangle}\,. \tag{4.13}$$

The expressions (4.8) and (4.13) satisfy the consistency check,

$$A_6^{\rm tr}(\Lambda_1^-, g_2^-, g_3^-, g_4^+, \bar\Lambda_5^+, g_6^+) = \left[A_6^{\rm tr}(g_1^-, \Lambda_2^-, g_3^-, \bar\Lambda_4^+, g_5^+, g_6^+)\right]\Big|_{j\to j+3}\,. \tag{4.14}$$

Thus we have a self-consistent set of six point, two gluino tree amplitudes for the helicity configuration $(- - - + + +)$.

Next we consider the helicity configuration $(--+-++)$ and obtain two gluino amplitudes from the gluonic amplitude:

$$
\begin{aligned}
A_6^{\rm tr}(g_1^-,g_2^-,g_3^+,g_4^-,g_5^+,g_6^+) = & \frac{i\,\langle 1\,2\rangle^3\,[5\,6]^3}{t_{123}\,\langle 2\,3\rangle\,[4\,5]\,\langle 4|K_{123}|1\rangle\langle 6|K_{123}|3\rangle} + \\
& + \frac{i\langle 3|K_{234}|1\rangle^4}{t_{234}\,[2\,3]\,[3\,4]\,\langle 5\,6\rangle\,\langle 6\,1\rangle\,\langle 2|K_{234}|5\rangle\langle 4|K_{234}|1\rangle} + \\
& + \frac{i\langle 6|K_{612}|4\rangle^4}{t_{345}\,[6\,1]\,[1\,2]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 6|K_{612}|3\rangle\langle 2|K_{612}|5\rangle}\,.
\end{aligned}
\tag{4.15}
$$

Six amplitudes involving two gluinos are needed to generate all possibilities by relabeling, conjugation and flipping:

$$
\begin{aligned}
& A_6^{\rm tr}(\Lambda_1^-,g_2^-,\bar\Lambda_3^+,g_4^-,g_5^+,g_6^+)\,, \quad A_6^{\rm tr}(g_1^-,\Lambda_2^-,\bar\Lambda_3^+,g_4^-,g_5^+,g_6^+)\,, \\
& A_6^{\rm tr}(g_1^-,g_2^-,\bar\Lambda_3^+,\Lambda_4^-,g_5^+,g_6^+)\,, \quad A_6^{\rm tr}(\Lambda_1^-,g_2^-,g_3^+,g_4^-,\bar\Lambda_5^+,g_6^+)\,, \\
& A_6^{\rm tr}(g_1^-,\Lambda_2^-,g_3^+,g_4^-,\bar\Lambda_5^+,g_6^+)\,, \quad A_6^{\rm tr}(\Lambda_1^-,g_2^-,g_3^+,g_4^-,g_5^+,\bar\Lambda_6^+)\,.
\end{aligned}
\tag{4.16}
$$

These are related to the gluonic amplitude via the three SWI,

$$
\begin{aligned}
0 = & \langle 1\,\eta\rangle\,A_6^{\rm tr}(\Lambda_1^-,g_2^-,\bar\Lambda_3^+,g_4^-,g_5^+,g_6^+) + \langle 2\,\eta\rangle\,A_6^{\rm tr}(g_1^-,\Lambda_2^-,\bar\Lambda_3^+,g_4^-,g_5^+,g_6^+) + \\
& + \langle 4\,\eta\rangle\,A_6^{\rm tr}(g_1^-,g_2^-,\bar\Lambda_3^+,\Lambda_4^-,g_5^+,g_6^+) - \langle 3\,\eta\rangle\,A_6^{\rm tr}(g_1^-,g_2^-,g_3^+,g_4^-,g_5^+,g_6^+)\,, \\
0 = & \langle 1\,\eta\rangle\,A_6^{\rm tr}(\Lambda_1^-,g_2^-,g_3^+,g_4^-,\bar\Lambda_5^+,g_6^+) + \langle 2\,\eta\rangle\,A_6^{\rm tr}(g_1^-,\Lambda_2^-,g_3^+,g_4^-,\bar\Lambda_5^+,g_6^+) + \\
& + \langle 4\,\eta\rangle\,A_6^{\rm tr}(g_1^-,g_2^-,g_3^+,\Lambda_4^-,\bar\Lambda_5^+,g_6^+) - \langle 5\,\eta\rangle\,A_6^{\rm tr}(g_1^-,g_2^-,g_3^+,g_4^-,g_5^+,g_6^+)\,, \\
0 = & \langle 1\,\eta\rangle\,A_6^{\rm tr}(\Lambda_1^-,g_2^-,g_3^+,g_4^-,g_5^+,\bar\Lambda_6^+) + \langle 2\,\eta\rangle\,A_6^{\rm tr}(g_1^-,\Lambda_2^-,g_3^+,g_4^-,g_5^+,\bar\Lambda_6^+) + \\
& + \langle 4\,\eta\rangle\,A_6^{\rm tr}(g_1^-,g_2^-,g_3^+,\Lambda_4^-,g_5^+,\bar\Lambda_6^+) - \langle 6\,\eta\rangle\,A_6^{\rm tr}(g_1^-,g_2^-,g_3^+,g_4^-,g_5^+,g_6^+)\,.
\end{aligned}
\tag{4.17}
$$

To solve the first of these, as before, we find two independent identities,

$$
\begin{aligned}
\langle 6|K_{612}|4\rangle\,\langle 3\,\eta\rangle &= \langle 6|K_{612}|3\rangle\,\langle 4\,\eta\rangle - [1\,6]\,\langle 3\,4\rangle\,\langle 1\,\eta\rangle - [2\,6]\,\langle 3\,4\rangle\,\langle 2\,\eta\rangle\,, \\
\langle 3\,\eta\rangle\,\langle 3|K_{234}|1\rangle &= t_{234}\,\langle 1\,\eta\rangle - \langle 2|K_{234}|1\rangle\,\langle 2\,\eta\rangle - \langle 4|K_{234}|1\rangle\,\langle 4\,\eta\rangle\,,
\end{aligned}
\tag{4.18}
$$

which give the following solutions to the SWI:

$$
\begin{aligned}
A_6^{\rm tr}(\Lambda_1^-,g_2^-,\bar\Lambda_3^+,g_4^-,g_5^+,g_6^+) &= [6\,1]\,\langle 3\,4\rangle\,X + t_{234}Y\,, \\
A_6^{\rm tr}(g_1^-,\Lambda_2^-,\bar\Lambda_3^+,g_4^-,g_5^+,g_6^+) &= [6\,2]\,\langle 3\,4\rangle\,X - \langle 2|K_{234}|1\rangle Y\,, \\
A_6^{\rm tr}(g_1^-,g_2^-,g_3^+,g_4^-,g_5^+,g_6^+) &= \langle 6|K_{612}|4\rangle X + \langle 3|K_{234}|1\rangle Y\,, \\
A_6^{\rm tr}(g_1^-,g_2^-,\bar\Lambda_3^+,\Lambda_4^-,g_5^+,g_6^+) &= \langle 6|K_{612}|3\rangle X - \langle 4|K_{234}|1\rangle Y\,.
\end{aligned}
\tag{4.19}
$$

We could rewrite the purely gluonic tree amplitude in the form $\langle 6|K_{612}|4\rangle X + \langle 3|K_{234}|1\rangle Y$ by using the identity,

$$
\frac{\langle 1\,2\rangle\,[5\,6]}{\langle 4|K_{234}|1\rangle\langle 6|K_{612}|3\rangle} = -\frac{\langle 3|K_{234}|1\rangle}{\langle 4|K_{234}|1\rangle\langle 2|K_{234}|5\rangle} + \frac{\langle 6|K_{612}|4\rangle}{\langle 6|K_{612}|3\rangle\langle 2|K_{612}|5\rangle}\,.
\tag{4.20}
$$

However, it is more convenient and in line with our philosophy of not generating extra poles to use the Schouten identity to produce,

$$\langle 3\,\eta\rangle\,\langle 1\,2\rangle\,[5\,6] = \langle 1\,\eta\rangle\,\langle 3\,2\rangle\,[5\,6] + \langle 2\,\eta\rangle\,\langle 1\,3\rangle\,[5\,6]\;. \tag{4.21}$$

Whether we rearrange to use two identities or use three, we obtain the same solutions,

$$\begin{aligned}
A_6^{\rm tr}(\Lambda_1^-, g_2^-, \bar\Lambda_3^+, g_4^-, g_5^+, g_6^+) &= \frac{-i\,\langle 1\,2\rangle^2\,\langle 2\,3\rangle\,[5\,6]^3}{t_{123}\,\langle 2\,3\rangle\,[4\,5]\,\langle 4|K_{123}|1\rangle\langle 6|K_{123}|3\rangle} + \\
&\quad + \frac{i\langle 3|K_{234}|1\rangle^3 t_{234}}{t_{234}\,[2\,3]\,[3\,4]\,\langle 5\,6\rangle\,\langle 6\,1\rangle\,\langle 2|K_{234}|5\rangle\langle 4|K_{234}|1\rangle} + \\
&\quad + \frac{i\langle 6|K_{612}|4\rangle^3\,[6\,1]\,\langle 3\,4\rangle}{t_{345}\,[6\,1]\,[1\,2]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 6|K_{612}|3\rangle\langle 2|K_{612}|5\rangle} \\
A_6^{\rm tr}(g_1^-, \Lambda_2^-, \bar\Lambda_3^+, g_4^-, g_5^+, g_6^+) &= \frac{i\,\langle 1\,2\rangle^2\,\langle 1\,3\rangle\,[5\,6]^3}{t_{123}\,\langle 2\,3\rangle\,[4\,5]\,\langle 4|K_{123}|1\rangle\langle 6|K_{123}|3\rangle} + \\
&\quad + \frac{-i\langle 3|K_{234}|1\rangle^3\langle 2|K_{234}|1\rangle}{t_{234}\,[2\,3]\,[3\,4]\,\langle 5\,6\rangle\,\langle 6\,1\rangle\,\langle 2|K_{234}|5\rangle\langle 4|K_{234}|1\rangle} + \\
&\quad + \frac{i\langle 6|K_{612}|4\rangle^3\,[6\,2]\,\langle 3\,4\rangle}{t_{345}\,[6\,1]\,[1\,2]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 6|K_{612}|3\rangle\langle 2|K_{612}|5\rangle} \\
A_6^{\rm tr}(g_1^-, g_2^-, \bar\Lambda_3^+, \Lambda_4^-, g_5^+, g_6^+) &= \frac{-i\langle 3|K_{234}|1\rangle^3\langle 4|K_{234}|1\rangle}{t_{234}\,[2\,3]\,[3\,4]\,\langle 5\,6\rangle\,\langle 6\,1\rangle\,\langle 2|K_{234}|5\rangle\langle 4|K_{234}|1\rangle} + \\
&\quad + \frac{i\langle 6|K_{612}|4\rangle^3\langle 6|K_{612}|3\rangle}{t_{345}\,[6\,1]\,[1\,2]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 6|K_{612}|3\rangle\langle 2|K_{612}|5\rangle}\,.
\end{aligned} \tag{4.22}$$

The remaining two amplitudes can be obtained similarly.

For the final gluonic configuration,

$$\begin{aligned}
A_6^{\rm tr}(g_1^-, g_2^+, g_3^-, g_4^+, g_5^-, g_6^+) &= \frac{i\langle 2|K_{123}|5\rangle^4}{t_{123}\,[1\,2]\,[2\,3]\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,\langle 1|K_{123}|4\rangle\langle 3|K_{123}|6\rangle} + \\
&\quad + \frac{i\langle 6|K_{234}|3\rangle^4}{t_{234}\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,[5\,6]\,[6\,1]\,\langle 5|K_{234}|2\rangle\langle 1|K_{234}|4\rangle} + \\
&\quad + \frac{i\langle 4|K_{345}|1\rangle^4}{t_{345}\,\langle 6\,1\rangle\,\langle 1\,2\rangle\,[3\,4]\,[4\,5]\,\langle 3|K_{345}|6\rangle\langle 5|K_{345}|2\rangle}\,,
\end{aligned} \tag{4.23}$$

there are two independent amplitudes involving two gluinos,

$$A_6^{\rm tr}(\Lambda_1^-, \bar\Lambda_2^+, g_3^-, g_4^+, g_5^-, g_6^+)\,, \qquad A_6^{\rm tr}(g_1^-, \bar\Lambda_2^+, g_3^-, g_4^+, \Lambda_5^-, g_6^+)\,, \tag{4.24}$$

which we can obtain from the SWI,

$$\begin{aligned}
0 &= \langle 1\,\eta\rangle\,A_6^{\rm tr}(\Lambda_1^-, \bar\Lambda_2^+, g_3^-, g_4^+, g_5^-, g_6^+) + \langle 3\,\eta\rangle\,A_6^{\rm tr}(g_1^-, \bar\Lambda_2^+, \Lambda_3^-, g_4^+, g_5^-, g_6^+) - \\
&\quad - \langle 2\,\eta\rangle\,A_6^{\rm tr}(g_1^-, g_2^+, g_3^-, g_4^+, g_5^-, g_6^+) + \langle 5\,\eta\rangle\,A_6^{\rm tr}(g_1^-, \bar\Lambda_2^+, g_3^-, g_4^+, \Lambda_5^-, g_6^+)\,.
\end{aligned} \tag{4.25}$$

We solve this using the identities,

$$\begin{aligned}
\langle 2|K_{123}|5\rangle\,\langle 2\,\eta\rangle &= t_{123}\,\langle 5\,\eta\rangle - \langle 1|K_{123}|5\rangle\,\langle 1\,\eta\rangle - \langle 3|K_{123}|5\rangle\,\langle 3\,\eta\rangle \\
\langle 6|K_{234}|3\rangle\,\langle 2\,\eta\rangle &= \langle 6|K_{234}|2\rangle\,\langle 3\,\eta\rangle + \langle 2\,3\rangle\,[5\,6]\,\langle 5\,\eta\rangle - \langle 2\,3\rangle\,[6\,1]\,\langle 1\,\eta\rangle \\
\langle 4|K_{345}|1\rangle\,\langle 2\,\eta\rangle &= \langle 4|K_{345}|2\rangle\,\langle 1\,\eta\rangle + [3\,4]\,\langle 1\,2\rangle\,\langle 3\,\eta\rangle - [4\,5]\,\langle 1\,2\rangle\,\langle 5\,\eta\rangle\,,
\end{aligned} \tag{4.26}$$

giving the tree amplitudes,

$$
\begin{aligned}
A_6^{\rm tr}(\Lambda_1^-,\bar\Lambda_2^+,g_3^-,g_4^+,g_5^-,g_6^+) = {} & \frac{-i\langle 2|K_{123}|5\rangle^3\langle 1|K_{123}|5\rangle}{t_{123}\,[1\,2]\,[2\,3]\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,\langle 1|K_{123}|4\rangle\langle 3|K_{123}|6\rangle}+\\
&+\frac{-i\langle 6|K_{234}|3\rangle^3\,\langle 2\,3\rangle\,[6\,1]}{t_{234}\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,[5\,6]\,[6\,1]\,\langle 5|K_{234}|2\rangle\langle 1|K_{234}|4\rangle}+\\
&+\frac{i\langle 4|K_{345}|1\rangle^3\langle 4|K_{345}|2\rangle}{t_{345}\,\langle 6\,1\rangle\,\langle 1\,2\rangle\,[3\,4]\,[4\,5]\,\langle 3|K_{345}|6\rangle\langle 5|K_{345}|2\rangle}\\
A_6^{\rm tr}(g_1^-,\bar\Lambda_2^+,\Lambda_3^-,g_4^+,g_5^-,g_6^+) = {} & \frac{-i\langle 2|K_{123}|5\rangle^3\langle 3|K_{123}|5\rangle}{t_{123}\,[1\,2]\,[2\,3]\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,\langle 1|K_{123}|4\rangle\langle 3|K_{123}|6\rangle}+\\
&+\frac{i\langle 6|K_{234}|3\rangle^3\langle 6|K_{234}|2\rangle}{t_{234}\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,[5\,6]\,[6\,1]\,\langle 5|K_{234}|2\rangle\langle 1|K_{234}|4\rangle}+\\
&+\frac{i\langle 4|K_{345}|1\rangle^3\,[3\,4]\,\langle 1\,2\rangle}{t_{345}\,\langle 6\,1\rangle\,\langle 1\,2\rangle\,[3\,4]\,[4\,5]\,\langle 3|K_{345}|6\rangle\langle 5|K_{345}|2\rangle}\\
A_6^{\rm tr}(g_1^-,\bar\Lambda_2^+,g_3^-,g_4^+,\Lambda_5^-,g_6^+) = {} & \frac{i\langle 2|K_{123}|5\rangle^3 t_{123}}{t_{123}\,[1\,2]\,[2\,3]\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,\langle 1|K_{123}|4\rangle\langle 3|K_{123}|6\rangle}+\\
&+\frac{i\langle 6|K_{234}|3\rangle^3\,\langle 2\,3\rangle\,[5\,6]}{t_{234}\,\langle 2\,3\rangle\,\langle 3\,4\rangle\,[5\,6]\,[6\,1]\,\langle 5|K_{234}|2\rangle\langle 1|K_{234}|4\rangle}+\\
&+\frac{-i\langle 4|K_{345}|1\rangle^3\,[4\,5]\,\langle 1\,2\rangle}{t_{345}\,\langle 6\,1\rangle\,\langle 1\,2\rangle\,[3\,4]\,[4\,5]\,\langle 3|K_{345}|6\rangle\langle 5|K_{345}|2\rangle}\,. \qquad (4.27)
\end{aligned}
$$

The six-point two-quark amplitudes have been computed previously [12] and can be obtained in compact expressions using recursion relations [30]. Our results for adjacent gluinos match these exactly — demonstrating that by respecting the symmetries and factorisation structures of the amplitudes one can use the SWI to generate the correct results.

5. Six point one loop NMHV amplitudes with two gluinos

The SWI apply to all orders in perturbation theory, so we can apply our technique to one-loop amplitudes. Furthermore, $\mathcal{N}=4$ one-loop amplitudes can be expressed as sums of box integrals with rational coefficients [20]. Since the box integrals are an independent set of functions the SWI for these amplitudes will apply box by box.

For the six-point, one loop, NMHV amplitudes the only types of box contributing are the "two-mass-hard" and one-mass boxes. These appear in certain very specific combinations:

$$
\begin{aligned}
W_6^{(i)} \equiv {} & F_i^{\rm 1m}+F_{i+3}^{\rm 1m}+F_{2;i+1}^{{\rm 2m}\,h}+F_{2;i+4}^{{\rm 2m}\,h}\\
= {} & -\frac{1}{2\epsilon^2}\sum_{j=1}^{6}\left(\frac{\mu^2}{-s_{j,j+1}}\right)^{\epsilon}-\ln\left(\frac{-t_{i,i+1,i+2}}{-s_{i,i+1}}\right)\ln\left(\frac{-t_{i,i+1,i+2}}{-s_{i+1,i+2}}\right)-\\
&-\ln\left(\frac{-t_{i,i+1,i+2}}{-s_{i+3,i+4}}\right)\ln\left(\frac{-t_{i,i+1,i+2}}{-s_{i+4,i+5}}\right)+\ln\left(\frac{-t_{i,i+1,i+2}}{-s_{i+2,i+3}}\right)\ln\left(\frac{-t_{i,i+1,i+2}}{-s_{i+5,i}}\right)+\\
&+\frac{1}{2}\ln\left(\frac{-s_{i,i+1}}{-s_{i+3,i+4}}\right)\ln\left(\frac{-s_{i+1,i+2}}{-s_{i+4,i+5}}\right)+\frac{1}{2}\ln\left(\frac{-s_{i-1,i}}{-s_{i,i+1}}\right)\ln\left(\frac{-s_{i+1,i+2}}{-s_{i+2,i+3}}\right)+
\end{aligned}
$$

$$+\frac{1}{2}\ln\left(\frac{-s_{i+2,i+3}}{-s_{i+3,i+4}}\right)\ln\left(\frac{-s_{i+4,i+5}}{-s_{i+5,i}}\right)+\frac{\pi^2}{3}\,. \tag{5.1}$$

There are only three independent $W_6^{(i)}$ since $W_6^{(i+3)} = W_6^{(i)}$. The $W_6^{(i)}$ have several rather special features which will extend to amplitudes involving fermions. Firstly, even though the integral functions individually contain dilogarithms, these drop out of the $W_6^{(i)}$. Secondly, the IR singularities take the rather simple form,

$$W_6^{(i)} = -\frac{3}{\epsilon^2}+\frac{1}{2}\sum_{j=1}^{6}\frac{\ln(-s_{j,j+1})}{\epsilon}+O(\epsilon^0)\,, \tag{5.2}$$

which leads to the sum of the coefficients of the $W_6^{(i)}$ being proportional to the tree amplitude.

The first set of amplitudes we shall consider are based on the gluonic amplitude,

$$A_6^{\mathcal{N}=4}(1^-,2^-,3^-,4^+,5^+,6^+) = c_\Gamma\left[B_1\,W_6^{(1)}+B_2\,W_6^{(2)}+B_3\,W_6^{(3)}\right], \tag{5.3}$$

where,

$$\begin{aligned}
B_1 &= B_0 \equiv i\frac{(t_{123})^3}{[1\,2]\,[2\,3]\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,\langle 1|K_{123}|4\rangle\langle 3|K_{123}|6\rangle}\,,\\
B_2 &= \left(\frac{\langle 4|K_{234}|1\rangle}{t_{234}}\right)^4 B_+ + \left(\frac{\langle 2\,3\rangle\,[5\,6]}{t_{234}}\right)^4 B_+^\dagger\,,\\
B_3 &= \left(\frac{\langle 6|K_{345}|3\rangle}{t_{345}}\right)^4 B_- + \left(\frac{\langle 1\,2\rangle\,[4\,5]}{t_{345}}\right)^4 B_-^\dagger\,,
\end{aligned} \tag{5.4}$$

and,

$$B_+ = B_0|_{j\to j+1}\,,\qquad B_- = B_0|_{j\to j-1}\,, \tag{5.5}$$

where the operation † implies $[i\,j]\leftrightarrow\langle j\,i\rangle$. This amplitude has two symmetries,

$$\begin{aligned}
S_1:&\;\; A_6^{\mathcal{N}=4}(1^-,2^-,3^-,4^+,5^+,6^+) = \left[A_6^{\mathcal{N}=4}(1^-,2^-,3^-,4^+,5^+,6^+)\right]^\dagger_{j\to j+3}\,,\\
S_2:&\;\; A_6^{\mathcal{N}=4}(1^-,2^-,3^-,4^+,5^+,6^+) = \left[A_6^{\mathcal{N}=4}(1^-,2^-,3^-,4^+,5^+,6^+)\right]^\dagger_{j\to 6-j}\,,
\end{aligned} \tag{5.6}$$

which impose constraints on the coefficients. Under S_1, $W_i \to W_i$ so we have,

$$S_1: B_i \longrightarrow B_i \tag{5.7}$$

whereas under S_2, $W_1\to W_1$ and $W_2\leftrightarrow W_3$ so that

$$S_2:\;\; B_1\longrightarrow B_1\,,\qquad B_2\leftrightarrow B_3\,. \tag{5.8}$$

The coefficients clearly satisfy these conditions when we note that B_0 itself satisfies,

$$S_1:\;\; B_0\longrightarrow B_0\,,\qquad S_2:\;\; B_0\longrightarrow B_0\,. \tag{5.9}$$

Applying S_i to the gluino amplitudes provides a set of consistency conditions that enable us to resolve the ambiguities that arise in solving the SWI.

JHEP08(2005)055

As for the tree amplitudes, we can generate all the possible two-gluino amplitudes from a minimal set of four by conjugation, relabeling and flipping. These gluino amplitudes have a subset of the invariances of the gluonic amplitudes. Specifically, $A(g_1^-, \Lambda_2^-, g_3^-, g_4^+, \bar{\Lambda}_5^+, g_6^+)$ is invariant under S_1 and S_2, while $A(\Lambda_1^-, g_2^-, g_3^-, \bar{\Lambda}_4^+, g_5^+, g_6^+)$ is only invariant under S_1, $A(g_1^-, g_2^-, \Lambda_3^-, \bar{\Lambda}_4^+, g_5^+, g_6^+)$ is only invariant under S_2 and $A(g_1^-, \Lambda_2^-, g_3^-, \bar{\Lambda}_4^+, g_5^+, g_6^+)$ is invariant under neither.

For this helicity configuration the SWI are (3.6) and (4.10). To solve for B_1 we need identities involving $\langle 4\,\eta\rangle$ and $\langle 5\,\eta\rangle$. These are,

$$\begin{aligned} t_{123}\,\langle 4\,\eta\rangle &= \langle 1|K_{123}|4\rangle\,\langle 1\,\eta\rangle + \langle 2|K_{123}|4\rangle\,\langle 2\,\eta\rangle + \langle 3|K_{123}|4\rangle\,\langle 3\,\eta\rangle\ , \\ t_{123}\,\langle 5\,\eta\rangle &= \langle 1|K_{123}|5\rangle\,\langle 1\,\eta\rangle + \langle 2|K_{123}|5\rangle\,\langle 2\,\eta\rangle + \langle 3|K_{123}|5\rangle\,\langle 3\,\eta\rangle\ . \end{aligned} \tag{5.10}$$

We can check that these equations are consistent with the symmetries S_i: if we have solutions,

$$\begin{aligned} A\,\langle 4\,\eta\rangle &= B\,\langle 1\,\eta\rangle + C\,\langle 2\,\eta\rangle + D\,\langle 3\,\eta\rangle\ , \\ A'\,\langle 5\,\eta\rangle &= B'\,\langle 1\,\eta\rangle + C'\,\langle 2\,\eta\rangle + D'\,\langle 3\,\eta\rangle\ , \end{aligned} \tag{5.11}$$

then we must have,

$$S_1:\ \left(\frac{B}{A}\right) \to \left(\frac{B}{A}\right),\quad S_2:\ \left(\frac{D}{A}\right) \to \left(\frac{D}{A}\right),\quad S_i:\ \left(\frac{C'}{A}\right) \to \left(\frac{C'}{A}\right). \tag{5.12}$$

The coefficients in (5.10) clearly satisfy these constraints. Thus we have solutions,

$$\begin{aligned} B_1(\Lambda_1^-, g_2^-, g_3^-, \bar{\Lambda}_4^+, g_5^+, g_6^+) &= \frac{i(t_{123})^2\langle 1|K_{123}|4\rangle}{[1\,2]\,[2\,3]\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,\langle 1|K_{123}|4\rangle\langle 3|K_{123}|6\rangle}\,, \\ B_1(g_1^-, \Lambda_2^-, g_3^-, \bar{\Lambda}_4^+, g_5^+, g_6^+) &= \frac{i(t_{123})^2\langle 2|K_{123}|4\rangle}{[1\,2]\,[2\,3]\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,\langle 1|K_{123}|4\rangle\langle 3|K_{123}|6\rangle}\,, \\ B_1(g_1^-, g_2^-, \Lambda_3^-, \bar{\Lambda}_4^+, g_5^+, g_6^+) &= \frac{i(t_{123})^2\langle 3|K_{123}|4\rangle}{[1\,2]\,[2\,3]\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,\langle 1|K_{123}|4\rangle\langle 3|K_{123}|6\rangle}\,, \\ B_1(g_1^-, \Lambda_2^-, g_3^-, g_4^+, \bar{\Lambda}_5^+, g_6^+) &= \frac{i(t_{123})^2\langle 2|K_{123}|5\rangle}{[1\,2]\,[2\,3]\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,\langle 1|K_{123}|4\rangle\langle 3|K_{123}|6\rangle}\,. \end{aligned} \tag{5.13}$$

To solve for the first three B_2's we use the identities,

$$\begin{aligned} \langle 4|K_{234}|1\rangle\,\langle 4\,\eta\rangle &= t_{234}\,\langle 1\,\eta\rangle - \langle 2|K_{234}|1\rangle\,\langle 2\,\eta\rangle - \langle 3|K_{234}|1\rangle\,\langle 3\,\eta\rangle\ , \\ \langle 2\,3\rangle\,\langle 4\,\eta\rangle &= \langle 4\,3\rangle\,\langle 2\,\eta\rangle + \langle 2\,4\rangle\,\langle 3\,\eta\rangle\ . \end{aligned} \tag{5.14}$$

Which give solutions,

$$B_2(\Lambda_1^-, g_2^-, g_3^-, \bar{\Lambda}_4^+, g_5^+, g_6^+) = \left(\frac{\langle 4|K_{234}|1\rangle^3}{t_{234}^3}\right) B_+\,, \tag{5.15}$$

$$\begin{aligned} B_2(g_1^-, \Lambda_2^-, g_3^-, \bar{\Lambda}_4^+, g_5^+, g_6^+) &= \left(\frac{-\langle 4|K_{234}|1\rangle^3\langle 2|K_{234}|1\rangle}{t_{234}^4}\right) B_+ + \left(\frac{\langle 2\,3\rangle^3\,\langle 4\,3\rangle\,[5\,6]^4}{t_{234}^4}\right) B_+^\dagger\,, \\ B_2(g_1^-, g_2^-, \Lambda_3^-, \bar{\Lambda}_4^+, g_5^+, g_6^+) &= \left(\frac{-\langle 4|K_{234}|1\rangle^3\langle 3|K_{234}|1\rangle}{t_{234}^4}\right) B_+ + \left(\frac{\langle 2\,3\rangle^3\,\langle 2\,4\rangle\,[5\,6]^4}{t_{234}^4}\right) B_+^\dagger\,. \end{aligned}$$

The absence of a second term from the first coefficient is consistent with the observation that this box-coefficient does not have a singlet term when we consider two-particle cuts in the t_{234} channel. (This observation would naturally lead us to an identity that does not involve $\langle 1\,\eta\rangle$)

For the final B_2 box coefficient there are three identities we might use:

$$\begin{aligned}
\langle 4|K_{234}|1\rangle \langle 5\,\eta\rangle &= \langle 4|K_{234}|5\rangle \langle 1\,\eta\rangle - [4\,2]\,\langle 1\,5\rangle \langle 2\,\eta\rangle - [4\,3]\,\langle 1\,5\rangle \langle 3\,\eta\rangle \,, \\
\langle 2\,3\rangle [5\,6]\,\langle 5\,\eta\rangle &= -\langle 2\,3\rangle [1\,6]\,\langle 1\,\eta\rangle + \langle 6|K_{234}|3\rangle \langle 2\,\eta\rangle - \langle 6|K_{234}|2\rangle \langle 3\,\eta\rangle \,, \\
\langle 2\,3\rangle \langle 5\,\eta\rangle &= \langle 5\,3\rangle \langle 2\,\eta\rangle + \langle 2\,5\rangle \langle 3\,\eta\rangle \,.
\end{aligned} \tag{5.16}$$

Of these, only the first two have the correct behavior under S_i. Using these identities we find,

$$B_2(g_1^-,\Lambda_2^-,g_3^-,g_4^+,\bar\Lambda_5^+,g_6^+) = \frac{-\langle 4|K_{234}|1\rangle^3\,[4\,2]\,\langle 1\,5\rangle}{t_{234}^4} B_+ + \frac{\langle 2\,3\rangle^3\,[5\,6]^3\,\langle 6|K_{234}|3\rangle}{t_{234}^4} B_+^\dagger \,, \tag{5.17}$$

which has the appropriate symmetries. This pair of identities also lead to the same forms for the other B_2 coefficients obtained previously.

For the B_3 coefficients, the identities,

$$\begin{aligned}
\langle 6|K_{345}|3\rangle \langle 4\,\eta\rangle &= [6\,1]\,\langle 3\,4\rangle \langle 1\,\eta\rangle + [6\,2]\,\langle 3\,4\rangle \langle 2\,\eta\rangle + \langle 6|K_{345}|4\rangle \langle 3\,\eta\rangle \,, \\
\langle 1\,2\rangle [4\,5]\,\langle 4\,\eta\rangle &= -\langle 5|K_{345}|2\rangle \langle 1\,\eta\rangle + \langle 5|K_{345}|1\rangle \langle 2\,\eta\rangle - \langle 1\,2\rangle [3\,5]\,\langle 3\,\eta\rangle \,, \\
\langle 6|K_{345}|3\rangle \langle 5\,\eta\rangle &= +[6\,1]\,\langle 3\,5\rangle \langle 1\,\eta\rangle + [6\,2]\,\langle 3\,5\rangle \langle 2\,\eta\rangle + \langle 6|K_{345}|5\rangle \langle 3\,\eta\rangle \,, \\
\langle 1\,2\rangle [4\,5]\,\langle 5\,\eta\rangle &= -\langle 1\,2\rangle [4\,3]\,\langle 3\,\eta\rangle + \langle 4|K_{345}|2\rangle \langle 1\,\eta\rangle - \langle 4|K_{345}|1\rangle \langle 2\,\eta\rangle \,,
\end{aligned} \tag{5.18}$$

give the following solutions with the correct symmetries under S_1,

$$\begin{aligned}
B_3(\Lambda_1^-,g_2^-,g_3^-,\bar\Lambda_4^+,g_5^+,g_6^+) &= \left(\frac{\langle 6|K_{345}|3\rangle^3\,\langle 3\,4\rangle\,[6\,1]}{t_{345}^4}\right) B_- + \\
&\quad + \left(\frac{-\langle 1\,2\rangle^3\,[4\,5]^3\,\langle 5|K_{345}|2\rangle}{t_{345}^4}\right) B_-^\dagger \,, \\
B_3(g_1^-,\Lambda_2^-,g_3^-,\bar\Lambda_4^+,g_5^+,g_6^+) &= \left(\frac{\langle 6|K_{345}|3\rangle^3\,\langle 3\,4\rangle\,[6\,2]}{t_{345}^4}\right) B_- + \\
&\quad + \left(\frac{\langle 1\,2\rangle^3\,[4\,5]^3\,\langle 5|K_{345}|1\rangle}{t_{345}^4}\right) B_-^\dagger \,, \\
B_3(g_1^-,g_2^-,\Lambda_3^-,\bar\Lambda_4^+,g_5^+,g_6^+) &= \left(\frac{\langle 6|K_{345}|3\rangle^3\langle 6|K_{345}|4\rangle}{t_{345}^4}\right) B_- + \\
&\quad + \left(\frac{-\langle 1\,2\rangle^4\,[4\,5]^3\,[3\,5]}{t_{345}^4}\right) B_-^\dagger \,, \\
B_3(g_1^-,\Lambda_2^-,g_3^-,g_4^+,\bar\Lambda_5^+,g_6^+) &= \left(\frac{\langle 6|K_{345}|3\rangle^3\,[6\,2]\,\langle 3\,5\rangle}{t_{345}^4}\right) B_- + \\
&\quad + \left(\frac{-\langle 1\,2\rangle^3\,[4\,5]^3\,\langle 4|K_{345}|1\rangle}{t_{345}^4}\right) B_-^\dagger \,.
\end{aligned} \tag{5.19}$$

Comparing these with the B_2 coefficients we see that the S_2 symmetry is also satisfied.

We can obtain the gluino amplitudes with helicity configurations $(--+-++)$ and $(-+-+-+)$ in a similar manner, i.e. by finding polynomial solutions to the SWI based on the gluonic amplitudes that respect the symmetries of the amplitudes. We have verified numerically that these expressions agree with those obtained using quadruple cuts (2.18). These coefficients are collected in the appendix and are available in Mathematica format to download.

There are straightforward relationships between the box coefficients and tree amplitudes. By necessity the IR divergences must be of the form,

$$\sim A^{\rm tree} \times \sum_i \frac{\ln(s_{ii+1})}{\epsilon} \,. \tag{5.20}$$

The box-coefficients must then satisfy,

$$B_1 + B_2 + B_3 = 2A^{\rm tree} \,. \tag{5.21}$$

It can be checked numerically that this is true by comparison with the tree amplitudes of section 4. The expressions for the tree amplitudes actually correspond to a subset of the terms comprising B_i. There are five terms in each $B_1 + B_2 + B_3$ expression. Two of these correspond exactly to the tree amplitudes of section 4 whereas the other three give an alternate, not trivially related, expression for the tree amplitude. For example taking the amplitude $A_6^{\rm tr}(g_1^-, g_2^-, \Lambda_3^-, \Lambda_4^+, g_5^+, g_6^+)$ we find,

$$\begin{aligned} A_6^{\rm tr} &= \frac{(t_{123})^3 \langle 3|K_{123}|4\rangle}{[1\,2]\,[2\,3]\,\langle 4\,5\rangle\,\langle 5\,6\rangle\,\langle 1^+|\not{K}|4^+\rangle\langle 3^+|\not{K}|6^+\rangle} + \left(\frac{\langle 2\,3\rangle^3\,\langle 2\,4\rangle\,[5\,6]^4}{t_{234}^4} \right) B_+^\dagger + \\ &\quad + \left(\frac{-\langle 1\,2\rangle^4\,[4\,5]^3\,[3\,5]}{t_{345}^4} \right) B_-^\dagger \,, \\ &= \left(\frac{-\langle 4^+|\not{K}|1^+\rangle^3 \langle 3^+|\not{K}|1^+\rangle}{t_{234}^4} \right) B_+ + \left(\frac{\langle 6^+|\not{K}|3^+\rangle^3 \langle 6^+|\not{K}|4^+\rangle}{t_{345}^4} \right) B_- \,, \end{aligned} \tag{5.22}$$

the second expression being that for the tree amplitude of section 4 and reference [30]. These relationships mirror very closely the behaviour of the gluon scattering amplitudes: it was the observation that the box coefficients reproduced the tree amplitudes in simple compact forms [4, 50] that led to the recursion relationships for tree amplitudes [28].

The twistor structure of the box coefficients is also rather simple: all the box-coefficients satisfy coplanarity constraints,

$$K_{abcd} B_i = 0 \,. \tag{5.23}$$

In fact this is satisfied by each of the terms within B_i individually.

6. Amplitudes with more than two fermions

We can use the SWI to obtain amplitudes involving four or more gluinos of the same flavour from those involving two gluinos. In the six-point case the tree amplitudes involving four and six fermions have been computed directly [51, 52] and also using recursion relations [53].

JHEP08(2005)055

If we consider n-point NMHV amplitudes with negative helicities on legs m_i, applying the $\mathcal{N}=1$ supersymmetry operator to,

$$A_n(g_1^+,\ldots,g_{m_1}^-,\ldots g_{m_2}^-,\ldots\Lambda_{m_3}^-,\ldots\bar\Lambda_r^+\ldots\bar\Lambda_s^+\ldots,g_n^+)\,, \tag{6.1}$$

gives the SWI,

$$0=\langle m_1\,\eta\rangle\,A_n^{m_1,m_3;r,s}+\langle m_2\,\eta\rangle\,A_n^{m_2,m_3;r,s}-\langle r\,\eta\rangle\,A_n^{m_3;s}-\langle s\,\eta\rangle\,A_n^{m_3;r}\,, \tag{6.2}$$

where we define,

$$\begin{aligned} A_n^{m_2,m_3;r,s}&\equiv A_n(g_1^+,\ldots,g_{m_1}^-,\ldots\Lambda_{m_2}^-,\ldots,\Lambda_{m_3}^-,\ldots\bar\Lambda_r^+\ldots\bar\Lambda_s^+\ldots,g_n^+)\,,\\ A_n^{m_3;r}&\equiv A_n(g_1^+,\ldots,g_{m_1}^-,\ldots g_{m_2}^-,\ldots,\Lambda_{m_3}^-,\ldots\bar\Lambda_r^+\ldots g_s^+\ldots,g_n^+)\,. \end{aligned} \tag{6.3}$$

This rank two system can be used to solve for the four fermion amplitudes in terms of the amplitudes with two fermions. For example choosing $\eta=m_1$ gives,

$$A_n^{m_2,m_3;r,s}=\frac{\langle r\,m_1\rangle}{\langle m_2\,m_1\rangle}A_n^{m_3;r}+\frac{\langle s\,m_1\rangle}{\langle m_2\,m_1\rangle}A_n^{m_3;s}\,. \tag{6.4}$$

Since we have used the $\mathcal{N}=1$ SWI, all of the fermions in this amplitude have the same flavour.

To obtain amplitudes with six gluinos we apply the supersymmetry operator to,

$$A_n(g_1^+,\ldots,g_{m_1}^-,\ldots\Lambda_{m_2}^-,\Lambda_{m_3}^-,\ldots\bar\Lambda_r^+\ldots\bar\Lambda_s^+\ldots,\bar\Lambda_t^+\ldots g_n^+)\,, \tag{6.5}$$

giving the SWI,

$$0=\langle m_1\,\eta\rangle\,A_n^{m_1,m_2,m_3;r,s,t}-\langle r\,\eta\rangle\,A_n^{m_2,m_3;s,t}-\langle s\,\eta\rangle\,A_n^{m_2,m_3;r,t}-\langle t\,\eta\rangle\,A_n^{m_2,m_3;r,s}\,, \tag{6.6}$$

which allows us to express the six fermion amplitude in terms of four fermion amplitudes. For example, choosing $\eta=r$,

$$A_n^{m_1,m_2,m_3;r,s,t}=\frac{\langle s\,r\rangle}{\langle m_1\,r\rangle}A_n^{m_2,m_3;r,t}+\frac{\langle t\,r\rangle}{\langle m_1\,r\rangle}A_n^{m_2,m_3;r,s}\,. \tag{6.7}$$

Again the fermions are all of the same flavour. These relations are exact to all orders in perturbation theory in any supersymmetric theory.

For amplitudes involving two fermion flavours we must be precise about which theory we are describing and in particular whether our theory contains scalars. Supersymmetric amplitudes with two flavours of fermions must include at least one scalar. For $\mathcal{N}\geq 2$ (and indeed for $\mathcal{N}=1$ with adjoint matter) the fermions have Yukawa couplings to the scalars which simultaneously change both the flavour and the helicity of the fermions. Such Yukawa couplings do not contribute to tree amplitudes with two gluinos, but they can contribute to amplitudes with four gluinos of two different flavours.

In $\mathcal{N}=2$ we can generate a SWI by applying Q_2 to,

$$A_n^{\mathcal{N}=2}(g_1^+,\ldots,g_{m_1}^-,\ldots g_{m_2}^-,\ldots,\Lambda_{m_3}^{1-},\ldots\bar\Lambda_r^{1+}\ldots\bar\Lambda_s^{2+}\ldots,g_n^+)\,. \tag{6.8}$$

We obtain,

$$
\begin{aligned}
0 = {} & \langle m_1\,\eta\rangle\, A_n^{\mathcal{N}=2}(g_1^+,\ldots,\Lambda_{m_1}^{2-},\ldots g_{m_2}^-,\ldots,\Lambda_{m_3}^{1-},\ldots\bar\Lambda_r^{1+}\ldots\bar\Lambda_s^{2+}\ldots,g_n^+)\,+\\
& +\langle m_2\,\eta\rangle\, A_n^{\mathcal{N}=2}(g_1^+,\ldots,g_{m_1}^-,\ldots\Lambda_{m_2}^{2-},\ldots,\Lambda_{m_3}^{1-},\ldots\bar\Lambda_r^{1+}\ldots\bar\Lambda_s^{2+}\ldots,g_n^+)\,-\\
& -i\,\langle m_3\,\eta\rangle\, A_n^{\mathcal{N}=2}(g_1^+,\ldots,g_{m_1}^-,\ldots g_{m_2}^-,\ldots,\phi_{m_3}^-,\ldots\bar\Lambda_r^{1+}\ldots\bar\Lambda_s^{2+}\ldots,g_n^+)\,-\\
& -\langle s\,\eta\rangle\, A_n^{\mathcal{N}=2}(g_1^+,\ldots,g_{m_1}^-,\ldots g_{m_2}^-,\ldots,\Lambda_{m_3}^{1-},\ldots\bar\Lambda_r^{1+}\ldots g_s^+\ldots,g_n^+)\,,
\end{aligned}
\tag{6.9}
$$

which can be used to determine the two flavour, four fermion amplitude in terms of a two fermion amplitude we have already calculated and a scalar-fermion-fermion amplitude which we discuss in the next section.

7. Amplitudes involving scalars

As noted above, for $\mathcal{N}\geq 2$ the fermions have Yukawa couplings to the scalars which simultaneously change both the flavour and the helicity of the fermion. At tree level, this vertex implies that amplitudes of the form,

$$
A_n^{\rm tr}(\phi^-,\Lambda^{1+},\Lambda^{2+},g^\pm,\ldots,g^\pm)\,,
\tag{7.1}
$$

need not vanish. In fact, there are non-vanishing MHV tree amplitudes of this form as may be seen in the expression of [14]. These amplitudes will appear in the SWI and must not be discarded.

In an $\mathcal{N}=2$ theory there are two flavours of gluino, Λ^i. Acting with Q_2 on,

$$
A_n^{\mathcal{N}=2}(\Lambda_1^{1-},g_2^-,g_3^-,\phi_4^+,g_5^+,\ldots,g_n^+)\,,
\tag{7.2}
$$

gives,

$$
\begin{aligned}
0 = {} & -i\,\langle 1\,\eta\rangle\, A_n^{\mathcal{N}=2}(\phi_1^-,g_2^-,g_3^-,\phi_4^+,g_5^+,\ldots,g_n^+)\,+\\
& +\langle 2\,\eta\rangle\, A_n^{\mathcal{N}=2}(\Lambda_1^{1-},\Lambda_2^{2-},g_3^-,\phi_4^+,g_5^+,\ldots,g_n^+)\,+\\
& +\langle 3\,\eta\rangle\, A_n^{\mathcal{N}=2}(\Lambda_1^{1-},g_2^-,\Lambda_3^{2-},\phi_4^+,g_5^+,\ldots,g_n^+)\,+\\
& +i\,\langle 4\,\eta\rangle\, A_n^{\mathcal{N}=2}(\Lambda_1^{1-},g_2^-,g_3^-,\bar\Lambda_4^{1+},g_5^+,\ldots,g_n^+)\,.
\end{aligned}
\tag{7.3}
$$

To solve this we need to find polynomial expressions of the form,

$$
0 = iA\,\langle 1\,\eta\rangle + B\,\langle 2\,\eta\rangle + C\,\langle 3\,\eta\rangle - iD\,\langle 4\,\eta\rangle\ .
\tag{7.4}
$$

Given such solutions, there will be relationships between the *individual terms* of the two gluino and two scalar amplitudes of the form,

$$
A_n^{\rm term}(\phi_1^-,g_2^-,g_3^-,\phi_4^+,g_5^+,\ldots,g_n^+) = \left(\frac{A}{D}\right) A_n^{\rm term}(\Lambda_1^{1-},g_2^-,g_3^-,\bar\Lambda_4^{1+},g_5^+,\ldots,g_n)\,.
\tag{7.5}
$$

If the appropriate solutions to (7.4) are the same as those used to obtain the two-gluino amplitudes, then the scalar terms will be of the form,

$$
A_n^{\rm term}(\phi_1^-,g_2^-,g_3^-,\phi_4^+,g_5^+,\ldots,g_n^+) = \left(\frac{A}{D}\right)^2 A_n^{\rm term}(g_1^-,g_2^-,g_3^-,g_4^+,g_5^+,\ldots,g_n)\,.
\tag{7.6}
$$

For gluonic amplitudes of the form,

$$A_n^{gluon} = \sum_i X_i\,, \tag{7.7}$$

we might expect amplitudes containing a pair of particles of spin h to have the form,

$$A_n^{h-\mathrm{pair}} = \sum_i (a_i)^{2-2h} X_i\,, \tag{7.8}$$

where $h = 1$ for gluons, $h = 1/2$ for fermions and $h = 0$ for scalars. Such structures are apparent in tree amplitudes as can be seen in the results of [30, 10]. For example, we can generalise our two gluino tree amplitude for the helicity configuration $(- - - + + +)$ to give,

$$\begin{aligned} A_6^{\mathcal{N}=2}(H_1^-, g_2^-, g_3^-, H_4^+, g_5^+, g_6^+) = {}& \left(\frac{t_{234}}{\langle 4|K_{234}|1\rangle}\right)^{2-2h} \frac{i\langle 4|K_{234}|1\rangle^3}{t_{234}\,[2\,3]\,[3\,4]\,\langle 5\,6\rangle\,\langle 6\,1\rangle\,\langle 2|K_{234}|5\rangle} + \\ & + \left(\frac{[1\,6]\,\langle 3\,4\rangle}{\langle 6|K_{612}|3\rangle}\right)^{2-2h} \frac{i\langle 6|K_{612}|3\rangle^3}{t_{612}\,[6\,1]\,[1\,2]\,\langle 3\,4\rangle\,\langle 4\,5\rangle\,\langle 2|K_{612}|5\rangle}\,, \end{aligned} \tag{7.9}$$

where H represent a gluon for $h = 1$, a gluino for $h = 1/2$, a scalar for $h = 0$ and an anti-gluino for $h = -1/2$. Such formulae are extremely useful when computing one-loop amplitudes using cuts (see for example [9, 10]).

This behaviour extends to the coefficients of the one-loop box functions and we give expressions for the box-functions for two scalars in the appendix. We have checked numerically for a representative sample that the box-coefficients thus obtained match those obtained via quadruple cuts.

Once we have the two gluino and two scalar amplitudes, the SWI (7.3) gives amplitudes such as,

$$A_n(\Lambda_1^{1-}, \Lambda_2^{2-}, g_3^-, \phi_4^+, g_5^+, g_6^+)\,, \tag{7.10}$$

directly. Given these amplitudes, the two flavour, four gluino amplitudes can be obtained directly from (6.9).

8. Conclusions

Recently there has been much progress in computing one-loop amplitudes with external gluons in compact forms, where the factorisation structure is simple and manifest. In this paper we have shown how to use Supersymmetric Ward Identities, together with the inherent symmetries of the amplitude, to generate one-loop amplitudes where the external particles are gluinos or adjoint scalars from these compact gluonic expressions. In particular, we have calculated all the six-point $\mathcal{N} = 4$ NMHV amplitudes involving two gluinos or scalars. The amplitudes with four or six gluinos (of a single flavour) have been given as linear combinations of the two gluino amplitudes. Although these results are specific to supersymmetric theories and adjoint fermions, they do reduce the amount of computation required to obtain results in non-supersymmetric theories with fundamental quarks. We also expect SWI to facilitate the calculation of loop scattering amplitudes in supergravity theories (see appendix B).

Organising Yang-Mills amplitudes in terms of helicity structure, particle type, colour and supersymmetry has helped enormously in understanding the structure of interactions in Yang-Mills theory and is the framework within which progress has occurred. However, phenomenologically, most of the quantum numbers which organise the amplitude are not observable in collider experiments. Consequently, the list of simple amplitudes required to compute an experimental quantity is rather long. The use of symmetries, such as SWI, to generate amplitudes without explicit computation is very helpful in this context.

Acknowledgments

We thank Emil Bjerrum-Bohr and Harald Ita for useful discussions. This work was supported by a PPARC rolling grant. SB would like to thank PPARC for a research studentship.

A. Summary of one-loop two gluino and two scalar six-point amplitudes in $\mathcal{N}=4$ SYM

The amplitudes for the $\mathcal{N}=4$ theory are all of the form,

$$A_6^{\mathcal{N}=4}(1,2,3,4,5,6) = c_\Gamma \left[c_1\, W_6^{(1)} + c_2\, W_6^{(2)} + c_3\, W_6^{(3)} \right], \tag{A.1}$$

with the coefficients c_i depending on the helicity and type of the six particles. This combination of box functions is given explicitly in eq. (5.1). The amplitudes will have one particle denoted H and a second denoted by $\bar{H}$. Again, H will denote either a scalar or $\Lambda^\pm$. The amplitudes are obtained using the specific values of h as defined in table 1.

H	$\bar{H}$	h
g^-	g^+	1
Λ^-	$\bar{\Lambda}^+$	$1/2$
ϕ^-	ϕ^+	0
Λ^+	$\bar{\Lambda}^-$	$-1/2$

Table 1: The values of h for the choices of external particle H.

We express the box coefficients in terms of B_0 and $B_\pm$ and their conjugates where,

$$B_0 = i\frac{(t_{123})^3}{[1\,2]\,[2\,3]\,\langle 4\,5\rangle\,\langle 5\,6\rangle\;\langle 1^+|\not{K}|4^+\rangle\langle 3^+|\not{K}|6^+\rangle}\,, \tag{A.2}$$

and

$$B_+ = B_0|_{j\to j+1}\,, \qquad B_- = B_0|_{j\to j-1}\,. \tag{A.3}$$

For amplitudes with helicity configuration $(---+++)$ we denote the c_i in the purely gluonic case by B_i,

$$A_6^{N=4}(1^-,2^-,3^-,4^+,5^+,6^+) = c_\Gamma \left[B_1\, W_6^{(1)} + B_2\, W_6^{(2)} + B_3\, W_6^{(3)} \right], \tag{A.4}$$

where,

$$\begin{aligned}
B_1 &= B_0\,,\\
B_2 &\equiv B_2^A + B_2^B = \left(\frac{\langle 4|K_{234}|1\rangle}{t_{234}}\right)^4 B_+ + \left(\frac{\langle 2\,3\rangle\,[5\,6]}{t_{234}}\right)^4 B_+^\dagger\,,\\
B_3 &\equiv B_3^A + B_3^B = \left(\frac{\langle 6|K_{345}|3\rangle}{t_{345}}\right)^4 B_- + \left(\frac{\langle 1\,2\rangle\,[4\,5]}{t_{345}}\right)^4 B_-^\dagger\,.
\end{aligned} \tag{A.5}$$

JHEP08(2005)055

For ease of presentation we shall denote the box-coefficients with fermions/scalars as B_i^{ab} when legs a and b are the H and $\bar{H}$ particles. The solutions for the B_i^{ab} for gluinos were derived in section 5 and we present them here again in a form that also gives the two scalar amplitudes. For the four independent configurations, $(ab) = (14)$, (24), (34) or (25), we find,

$$
\begin{aligned}
B_1^{14} &= \left(\frac{\langle 1|K_{123}|4\rangle}{t_{123}}\right)^{2-2h} B_0\,,\\
B_1^{24} &= \left(\frac{\langle 2|K_{123}|4\rangle}{t_{123}}\right)^{2-2h} B_0\,,\\
B_1^{34} &= \left(\frac{\langle 3|K_{123}|4\rangle}{t_{123}}\right)^{2-2h} B_0\,,\\
B_1^{25} &= \left(\frac{\langle 2|K_{123}|5\rangle}{t_{123}}\right)^{2-2h} B_0\,,
\end{aligned}
\tag{A.6}
$$

$$
\begin{aligned}
B_2^{14} &= \left(\frac{t_{234}}{\langle 4|K_{234}|1\rangle}\right)^{2-2h} B_2^A\,,\\
B_2^{24} &= \left(\frac{-\langle 2|K_{234}|1\rangle}{\langle 4|K_{234}|1\rangle}\right)^{2-2h} B_2^A + \left(\frac{\langle 4\,3\rangle}{\langle 2\,3\rangle}\right)^{2-2h} B_2^B\,,\\
B_2^{34} &= \left(\frac{-\langle 3|K_{234}|1\rangle}{\langle 4|K_{234}|1\rangle}\right)^{2-2h} B_2^A + \left(\frac{\langle 2\,4\rangle}{\langle 2\,3\rangle}\right)^{2-2h} B_2^B\,,\\
B_2^{25} &= \left(\frac{-\,[4\,2]\,\langle 1\,5\rangle}{\langle 4|K_{234}|1\rangle}\right)^{2-2h} B_2^A + \left(\frac{\langle 6|K_{234}|3\rangle}{\langle 2\,3\rangle\,[5\,6]}\right)^{2-2h} B_2^B\,,
\end{aligned}
\tag{A.7}
$$

$$
\begin{aligned}
B_3^{14} &= \left(\frac{\langle 3\,4\rangle\,[6\,1]}{\langle 6|K_{345}|3\rangle}\right)^{2-2h} B_3^A + \left(\frac{-\langle 5|K_{345}|2\rangle}{\langle 1\,2\rangle\,[4\,5]}\right)^{2-2h} B_3^B\,,\\
B_3^{24} &= \left(\frac{\langle 3\,4\rangle\,[6\,2]}{\langle 6|K_{345}|3\rangle}\right)^{2-2h} B_3^A + \left(\frac{\langle 5|K_{345}|1\rangle}{\langle 1\,2\rangle\,[4\,5]}\right)^{2-2h} B_3^B\,,\\
B_3^{34} &= \left(\frac{\langle 6|K_{345}|4\rangle}{\langle 6|K_{345}|3\rangle}\right)^{2-2h} B_3^A + \left(-\frac{[3\,5]}{[4\,5]}\right)^{2-2h} B_3^B\,,\\
B_3^{25} &= \left(\frac{[6\,2]\,\langle 3\,5\rangle}{\langle 6|K_{345}|3\rangle}\right)^{2-2h} B_3^A + \left(\frac{-\langle 4|K_{345}|1\rangle}{\langle 1\,2\rangle\,[4\,5]}\right)^{2-2h} B_3^B\,.
\end{aligned}
\tag{A.8}
$$

Next we have amplitudes with helicity structure $(--+-++)$. In the purely gluonic case, the amplitude is symmetric under,

$$
S_1 : A_6^{\mathcal{N}=4}(1,2,3,4,5,6) \longrightarrow [A_6^{\mathcal{N}=4}(6,5,4,3,2,1)]^\dagger\,.
\tag{A.9}
$$

In this case we denote the coefficients of the $W_6^{(i)}$ by D_i. These are given by,

$$
\begin{aligned}
D_1 \equiv D_1^A + D_1^B &= \left(\frac{\langle 3|K_{123}|4\rangle}{t_{123}}\right)^4 B_0 + \left(\frac{\langle 1\,2\rangle\,[5\,6]}{t_{123}}\right)^4 B_0^\dagger\,,\\
D_2 \equiv D_2^A + D_2^B &= \left(\frac{\langle 3|K_{234}|1\rangle}{t_{234}}\right)^4 B_+ + \left(\frac{\langle 2\,4\rangle\,[5\,6]}{t_{234}}\right)^4 B_+^\dagger\,,
\end{aligned}
$$

$$D_3 \equiv D_3^A + D_3^B = \left(\frac{\langle 6|K_{345}|4\rangle}{t_{345}}\right)^4 B_- + \left(\frac{\langle 1\,2\rangle\,[3\,5]}{t_{345}}\right)^4 B_-^\dagger \,. \tag{A.10}$$

As above, we denote the coefficients of amplitudes with particle a of type H and particle b of type $\bar{H}$ by D_i^{ab}. For this helicity configuration there are six independent possibilities:

$$(ab) = (13), (23), (43), (15), (25), (16)\,. \tag{A.11}$$

These six box-coefficients are constrained by the system of three SWI (4.17). In solving these we must find solutions which satisfy,

$$\begin{aligned}
&S_1: \;\; D_1^{ab} \longrightarrow D_1^{ab} \quad (ab) = (34), (25), (16)\,,\\
&S_1: \;\; D_2^{ab} \leftrightarrow D_3^{ab} \quad (ab) = (34), (25), (16)\,.
\end{aligned} \tag{A.12}$$

The identities that give amplitudes with the appropriate symmetries are:

$$\begin{aligned}
\langle 3|K_{123}|4\rangle\,\langle 3\,\eta\rangle &= t_{123}\,\langle 4\,\eta\rangle - \langle 1|K_{123}|4\rangle\,\langle 1\,\eta\rangle - \langle 2|K_{123}|4\rangle\,\langle 2\,\eta\rangle\;,\\
\langle 1\,2\rangle\,[5\,6]\,\langle 3\,\eta\rangle &= \langle 1\,3\rangle\,[5\,6]\,\langle 2\,\eta\rangle + \langle 3\,2\rangle\,[5\,6]\,\langle 1\,\eta\rangle\;,\\
\langle 3|K_{234}|1\rangle\,\langle 3\,\eta\rangle &= t_{234}\,\langle 1\,\eta\rangle - \langle 2|K_{234}|1\rangle\,\langle 2\,\eta\rangle - \langle 4|K_{234}|1\rangle\,\langle 4\,\eta\rangle\;,\\
\langle 2\,4\rangle\,[5\,6]\,\langle 3\,\eta\rangle &= \langle 3\,4\rangle\,[5\,6]\,\langle 2\,\eta\rangle + \langle 2\,3\rangle\,[5\,6]\,\langle 4\,\eta\rangle\;,\\
\langle 6|K_{345}|4\rangle\,\langle 3\,\eta\rangle &= \langle 4\,3\rangle\,[6\,1]\,\langle 1\,\eta\rangle + \langle 4\,3\rangle\,[6\,2]\,\langle 2\,\eta\rangle + \langle 6|K_{345}|3\rangle\,\langle 4\,\eta\rangle\;,\\
\langle 1\,2\rangle\,[3\,5]\,\langle 3\,\eta\rangle &= -\langle 5|K_{345}|2\rangle\,\langle 1\,\eta\rangle + \langle 5|K_{345}|1\rangle\,\langle 2\,\eta\rangle - \langle 1\,2\rangle\,[4\,5]\,\langle 4\,\eta\rangle\;,
\end{aligned} \tag{A.13}$$

$$\begin{aligned}
\langle 3|K_{123}|4\rangle\,\langle 5\,\eta\rangle &= \langle 3|K_{123}|5\rangle\,\langle 4\,\eta\rangle - [3\,1]\,\langle 4\,5\rangle\,\langle 1\,\eta\rangle - [3\,2]\,\langle 4\,5\rangle\,\langle 2\,\eta\rangle\;,\\
\langle 1\,2\rangle\,[5\,6]\,\langle 5\,\eta\rangle &= -\,\langle 1\,2\rangle\,[4\,6]\,\langle 4\,\eta\rangle - \langle 6|K_{123}|1\rangle\,\langle 2\,\eta\rangle + \langle 6|K_{123}|2\rangle\,\langle 1\,\eta\rangle\;,\\
\langle 3|K_{234}|1\rangle\,\langle 5\,\eta\rangle &= \langle 3|K_{234}|5\rangle\,\langle 1\,\eta\rangle - \langle 1\,5\rangle\,[3\,2]\,\langle 2\,\eta\rangle - \langle 1\,5\rangle\,[3\,4]\,\langle 4\,\eta\rangle\;,\\
\langle 2\,4\rangle\,[5\,6]\,\langle 5\,\eta\rangle &= -\,\langle 2\,4\rangle\,[1\,6]\,\langle 1\,\eta\rangle - \langle 6|K_{234}|2\rangle\,\langle 4\,\eta\rangle + \langle 6|K_{234}|4\rangle\,\langle 2\,\eta\rangle\;,\\
\langle 6|K_{345}|4\rangle\,\langle 5\,\eta\rangle &= \langle 6|K_{345}|5\rangle\,\langle 4\,\eta\rangle + \langle 4\,5\rangle\,[6\,1]\,\langle 1\,\eta\rangle + \langle 4\,5\rangle\,[6\,2]\,\langle 2\,\eta\rangle\;,\\
\langle 1\,2\rangle\,[3\,5]\,\langle 5\,\eta\rangle &= -\,\langle 1\,2\rangle\,[3\,4]\,\langle 4\,\eta\rangle - \langle 3|K_{345}|1\rangle\,\langle 2\,\eta\rangle + \langle 3|K_{345}|2\rangle\,\langle 1\,\eta\rangle\;,
\end{aligned} \tag{A.14}$$

$$\begin{aligned}
\langle 3|K_{123}|4\rangle\,\langle 6\,\eta\rangle &= \langle 3|K_{123}|6\rangle\,\langle 4\,\eta\rangle - [3\,1]\,\langle 4\,6\rangle\,\langle 1\,\eta\rangle - [3\,2]\,\langle 4\,6\rangle\,\langle 2\,\eta\rangle\;,\\
\langle 1\,2\rangle\,[5\,6]\,\langle 6\,\eta\rangle &= \langle 1\,2\rangle\,[4\,5]\,\langle 4\,\eta\rangle + \langle 5|K_{123}|1\rangle\,\langle 2\,\eta\rangle - \langle 5|K_{123}|2\rangle\,\langle 1\,\eta\rangle\;,\\
\langle 3|K_{234}|1\rangle\,\langle 6\,\eta\rangle &= \langle 3|K_{234}|6\rangle\,\langle 1\,\eta\rangle - \langle 1\,6\rangle\,[3\,2]\,\langle 2\,\eta\rangle - \langle 1\,6\rangle\,[3\,4]\,\langle 4\,\eta\rangle\;,\\
\langle 2\,4\rangle\,[5\,6]\,\langle 6\,\eta\rangle &= \langle 2\,4\rangle\,[1\,5]\,\langle 1\,\eta\rangle + \langle 5|K_{234}|2\rangle\,\langle 4\,\eta\rangle - \langle 5|K_{234}|4\rangle\,\langle 2\,\eta\rangle\;,\\
\langle 6|K_{345}|4\rangle\,\langle 6\,\eta\rangle &= -t_{345}\,\langle 4\,\eta\rangle - \langle 1|K_{345}|4\rangle\,\langle 1\,\eta\rangle - \langle 2|K_{345}|4\rangle\,\langle 2\,\eta\rangle\;,\\
\langle 1\,2\rangle\,[3\,5]\,\langle 6\,\eta\rangle &= -\,[3\,5]\,\langle 6\,1\rangle\,\langle 2\,\eta\rangle - [3\,5]\,\langle 2\,6\rangle\,\langle 1\,\eta\rangle\;.
\end{aligned} \tag{A.15}$$

The box coefficients are then given by,

$$\begin{aligned}
D_1^{13} &= \left(-\frac{\langle 1|K_{123}|4\rangle}{\langle 3|K_{123}|4\rangle}\right)^{2-2h} D_1^A + \left(\frac{\langle 3\,2\rangle}{\langle 1\,2\rangle}\right)^{2-2h} D_1^B\,,\\
D_1^{23} &= \left(-\frac{\langle 2|K_{123}|4\rangle}{\langle 3|K_{123}|4\rangle}\right)^{2-2h} D_1^A + \left(\frac{\langle 1\,3\rangle}{\langle 1\,2\rangle}\right)^{2-2h} D_1^B\,,
\end{aligned}$$

JHEP08(2005)055

$$
\begin{aligned}
D_1^{43} &= \left(\frac{t_{123}}{\langle 3|K_{123}|4\rangle}\right)^{2-2h} D_1^A\,,\\
D_1^{15} &= \left(\frac{\langle 4\,5\rangle\,[1\,3]}{\langle 3|K_{123}|4\rangle}\right)^{2-2h} D_1^A + \left(\frac{\langle 6|K_{123}|2\rangle}{\langle 1\,2\rangle\,[5\,6]}\right)^{2-2h} D_1^B\,,\\
D_1^{25} &= \left(\frac{\langle 4\,5\rangle\,[2\,3]}{\langle 3|K_{123}|4\rangle}\right)^{2-2h} D_1^A + \left(-\frac{\langle 6|K_{123}|1\rangle}{\langle 1\,2\rangle\,[5\,6]}\right)^{2-2h} D_1^B\,,\\
D_1^{16} &= \left(-\frac{[3\,1]\,\langle 4\,6\rangle}{\langle 3|K_{123}|4\rangle}\right)^{2-2h} D_1^A + \left(\frac{-\langle 5|K_{123}|2\rangle}{\langle 1\,2\rangle\,[5\,6]}\right)^{2-2h} D_1^B\,,
\end{aligned}
\tag{A.16}
$$

$$
\begin{aligned}
D_2^{13} &= \left(\frac{t_{234}}{\langle 3|K_{234}|1\rangle}\right)^{2-2h} D_2^A\,,\\
D_2^{23} &= \left(-\frac{\langle 2|K_{234}|1\rangle}{\langle 3|K_{234}|1\rangle}\right)^{2-2h} D_2^A + \left(\frac{\langle 3\,4\rangle}{\langle 2\,4\rangle}\right)^{2-2h} D_2^B\,,\\
D_2^{43} &= \left(-\frac{\langle 4|K_{234}|1\rangle}{\langle 3|K_{234}|1\rangle}\right)^{2-2h} D_2^A + \left(\frac{\langle 2\,3\rangle}{\langle 2\,4\rangle}\right)^{2-2h} D_2^B\,,\\
D_2^{15} &= \left(\frac{\langle 3|K_{234}|5\rangle}{\langle 3|K_{234}|1\rangle}\right)^{2-2h} D_2^A + \left(-\frac{[1\,6]}{[5\,6]}\right)^{2-2h} D_2^B\,,\\
D_2^{25} &= \left(\frac{[2\,3]\,\langle 1\,5\rangle}{\langle 3|K_{234}|1\rangle}\right)^{2-2h} D_2^A + \left(\frac{\langle 6|K_{234}|4\rangle}{\langle 2\,4\rangle\,[5\,6]}\right)^{2-2h} D_2^B\,,\\
D_2^{16} &= \left(\frac{\langle 3|K_{234}|6\rangle}{\langle 3|K_{234}|1\rangle}\right)^{2-2h} D_2^A + \left(-\frac{[5\,1]}{[5\,6]}\right)^{2-2h} D_2^B\,,
\end{aligned}
\tag{A.17}
$$

$$
\begin{aligned}
D_3^{13} &= \left(-\frac{[6\,1]\,\langle 3\,4\rangle}{\langle 6|K_{345}|4\rangle}\right)^{2-2h} D_3^A + \left(-\frac{\langle 5|K_{345}|2\rangle}{\langle 1\,2\rangle\,[3\,5]}\right)^{2-2h} D_3^B\,,\\
D_3^{23} &= \left(-\frac{[6\,2]\,\langle 3\,4\rangle}{\langle 6|K_{345}|4\rangle}\right)^{2-2h} D_3^A + \left(\frac{\langle 5|K_{345}|1\rangle}{\langle 1\,2\rangle\,[3\,5]}\right)^{2-2h} D_3^B\,,\\
D_3^{43} &= \left(\frac{\langle 6|K_{345}|3\rangle}{\langle 6|K_{345}|4\rangle}\right)^{2-2h} D_3^A + \left(-\frac{[4\,5]}{[3\,5]}\right)^{2-2h} D_3^B\,,\\
D_3^{15} &= \left(\frac{[6\,1]\,\langle 4\,5\rangle}{\langle 6|K_{345}|4\rangle}\right)^{2-2h} D_3^A + \left(\frac{\langle 3|K_{345}|2\rangle}{\langle 1\,2\rangle\,[3\,5]}\right)^{2-2h} D_3^B\,,\\
D_3^{25} &= \left(\frac{[6\,2]\,\langle 4\,5\rangle}{\langle 6|K_{345}|4\rangle}\right)^{2-2h} D_3^A + \left(-\frac{\langle 3|K_{345}|1\rangle}{\langle 1\,2\rangle\,[3\,5]}\right)^{2-2h} D_3^B\,,\\
D_3^{16} &= \left(-\frac{\langle 1|K_{345}|4\rangle}{\langle 6|K_{345}|4\rangle}\right)^{2-2h} D_3^A + \left(\frac{\langle 6\,2\rangle}{\langle 1\,2\rangle}\right)^{2-2h} D_3^B\,.
\end{aligned}
\tag{A.18}
$$

Next we have amplitudes with helicity structure $(-+-+-+)$. The purely gluonic amplitude is symmetric under,

$$
\begin{aligned}
S_1:&\quad A_6^{\mathcal{N}=4}(1,2,3,4,5,6) \longrightarrow A_6^{\mathcal{N}=4}(1,2,3,4,5,6)|_{j\to j+2}\,,\\
S_2:&\quad A_6^{\mathcal{N}=4}(1,2,3,4,5,6) \longrightarrow [A_6^{\mathcal{N}=4}(1,2,3,4,5,6)|_{j\to j+1}]^\dagger\,.
\end{aligned}
\tag{A.19}
$$

In this case we denote the coefficients of $W_6^{(i)}$ in the purely gluonic case by G_i. These are given by,

$$
\begin{aligned}
G_1 \equiv G_1^A + G_1^B &= \left(\frac{\langle 2|K_{123}|5\rangle}{t_{123}}\right)^4 B_0 + \left(\frac{\langle 1\,3\rangle\,[4\,6]}{t_{123}}\right)^4 B_0^\dagger\,, \\
G_2 \equiv G_2^A + G_2^B &= \left(\frac{\langle 6|K_{234}|3\rangle}{t_{234}}\right)^4 B_+^\dagger + \left(\frac{\langle 5\,1\rangle\,[2\,4]}{t_{234}}\right)^4 B_+\,, \\
G_3 \equiv G_3^A + G_3^B &= \left(\frac{\langle 4|K_{345}|1\rangle}{t_{345}}\right)^4 B_-^\dagger + \left(\frac{\langle 3\,5\rangle\,[6\,2]}{t_{345}}\right)^4 B_-\,.
\end{aligned}
\tag{A.20}
$$

Although there are only two independent configurations with two gluinos in this case, we present results for all the two gluino amplitudes appearing in the SWI (4.25). Amplitudes with the correct symmetries are produced by applying the following identities to the SWI:

$$
\begin{aligned}
\langle 2|K_{123}|5\rangle\,\langle 2\,\eta\rangle &= t_{123}\,\langle 5\,\eta\rangle - \langle 1|K_{123}|5\rangle\,\langle 1\,\eta\rangle - \langle 3|K_{123}|5\rangle\,\langle 3\,\eta\rangle\ , \\
\langle 1\,3\rangle\,[4\,6]\,\langle 2\,\eta\rangle &= \langle 2\,3\rangle\,[4\,6]\,\langle 1\,\eta\rangle + \langle 1\,2\rangle\,[4\,6]\,\langle 3\,\eta\rangle\ , \\
\langle 6|K_{234}|3\rangle\,\langle 2\,\eta\rangle &= -\,[6\,1]\,\langle 2\,3\rangle\,\langle 1\,\eta\rangle + \langle 6|K_{234}|2\rangle\,\langle 3\,\eta\rangle + [5\,6]\,\langle 2\,3\rangle\,\langle 5\,\eta\rangle\ , \\
\langle 5\,1\rangle\,[2\,4]\,\langle 2\,\eta\rangle &= -\,\langle 5\,1\rangle\,[3\,4]\,\langle 3\,\eta\rangle + \langle 4|K_{234}|5\rangle\,\langle 1\,\eta\rangle - \langle 4|K_{234}|1\rangle\,\langle 5\,\eta\rangle\ , \\
\langle 4|K_{345}|1\rangle\,\langle 2\,\eta\rangle &= \langle 4|K_{345}|2\rangle\,\langle 1\,\eta\rangle + [3\,4]\,\langle 1\,2\rangle\,\langle 3\,\eta\rangle - [4\,5]\,\langle 1\,2\rangle\,\langle 5\,\eta\rangle\ , \\
\langle 3\,5\rangle\,[6\,2]\,\langle 2\,\eta\rangle &= -\,\langle 3\,5\rangle\,[6\,1]\,\langle 1\,\eta\rangle - \langle 6|K_{345}|5\rangle\,\langle 3\,\eta\rangle + \langle 6|K_{345}|3\rangle\,\langle 5\,\eta\rangle\ .
\end{aligned}
\tag{A.21}
$$

The amplitudes are:

$$
\begin{aligned}
G_1^{12} &= \left(-\frac{\langle 1|K_{123}|5\rangle}{\langle 2|K_{123}|5\rangle}\right)^{2-2h} G_1^A + \left(\frac{\langle 2\,3\rangle}{\langle 1\,3\rangle}\right)^{2-2h} G_1^B\,, \\
G_1^{32} &= \left(-\frac{\langle 3|K_{123}|5\rangle}{\langle 2|K_{123}|5\rangle}\right)^{2-2h} G_1^A + \left(\frac{\langle 1\,2\rangle}{\langle 1\,3\rangle}\right)^{2-2h} G_1^B\,, \\
G_1^{52} &= \left(\frac{t_{123}}{\langle 2|K_{123}|5\rangle}\right)^{2-2h} G_1^A\,,
\end{aligned}
\tag{A.22}
$$

$$
\begin{aligned}
G_2^{12} &= \left(-\frac{\langle 2\,3\rangle\,[6\,1]}{\langle 6|K_{234}|3\rangle}\right)^{2-2h} G_2^A + \left(\frac{\langle 4|K_{234}|5\rangle}{\langle 5\,1\rangle\,[2\,4]}\right)^{2-2h} G_2^B\,, \\
G_2^{32} &= \left(\frac{\langle 6|K_{234}|2\rangle}{\langle 6|K_{234}|3\rangle}\right)^{2-2h} G_2^A + \left(-\frac{[3\,4]}{[2\,4]}\right)^{2-2h} G_2^B\,, \\
G_2^{52} &= \left(\frac{\langle 2\,3\rangle\,[5\,6]}{\langle 6|K_{234}|3\rangle}\right)^{2-2h} G_2^A + \left(-\frac{\langle 4|K_{234}|1\rangle}{\langle 5\,1\rangle\,[2\,4]}\right)^{2-2h} G_2^B\,,
\end{aligned}
\tag{A.23}
$$

$$
\begin{aligned}
G_3^{12} &= \left(\frac{\langle 4|K_{345}|2\rangle}{\langle 4|K_{345}|1\rangle}\right)^{2-2h} G_3^A + \left(-\frac{[6\,1]}{[6\,2]}\right)^{2-2h} G_3^B\,, \\
G_3^{32} &= \left(\frac{[3\,4]\,\langle 1\,2\rangle}{\langle 4|K_{345}|1\rangle}\right)^{2-2h} G_3^A + \left(-\frac{\langle 6|K_{345}|5\rangle}{\langle 3\,5\rangle\,[6\,2]}\right)^{2-2h} G_3^B\,, \\
G_3^{52} &= \left(-\frac{[4\,5]\,\langle 1\,2\rangle}{\langle 4|K_{345}|1\rangle}\right)^{2-2h} G_3^A + \left(\frac{\langle 6|K_{345}|3\rangle}{\langle 3\,5\rangle\,[6\,2]}\right)^{2-2h} G_3^B\,.
\end{aligned}
\tag{A.24}
$$

The six-point box-coefficients have been explicitly checked by numerically evaluating the quadruple cuts. The functional forms of the six-point coefficients B_i^{ab}, D_i^{ab} and G_i^{ab} are available in Mathematica format at: `http://pyweb.swan.ac.uk/~dunbar/SWAT430.html`.

B. Graviton scattering amplitudes

Some of the earliest applications of SWI were to graviton scattering amplitudes [11]. The MHV amplitudes involving different particle types obey relationships very similar to the Yang-Mills case,

$$M(g_1^-, H_2^-, H_3^+, g_4, \ldots, g_n^+) = \left(\frac{\langle 1\,3\rangle}{\langle 1\,2\rangle}\right)^{4-2h} M(g_1^-, g_2^-, g_3^+, g_4, \ldots, g_n^+)\,, \tag{B.1}$$

where h now runs over the helicities of the $\mathcal{N}=8$ supergravity multiplet, i.e. from $h=-2$ to $h=+2$.

If we solve the SWI for NMHV amplitudes, we might again expect to find amplitudes of the form,

$$\sum_i (X_i)^{4-2h} \times C_i\,. \tag{B.2}$$

Examination of the six-point NMHV tree amplitude [32, 33] reveals precisely this structure and it can also be found in the coefficients of loop amplitudes [54, 33, 55].

References

[1] E. Witten, *Perturbative gauge theory as a string theory in twistor space*, *Commun. Math. Phys.* **252** (2004) 189 [hep-th/0312171].

[2] F. Cachazo, P. Svrcek and E. Witten, *MHV vertices and tree amplitudes in gauge theory*, *JHEP* **09** (2004) 006 [hep-th/0403047].

[3] Z. Bern, V. Del Duca, L.J. Dixon and D.A. Kosower, *All non-maximally-helicity-violating one-loop seven-gluon amplitudes in $N=4$ super-Yang-Mills theory*, *Phys. Rev.* **D 71** (2005) 045006 [hep-th/0410224].

[4] Z. Bern, L.J. Dixon and D.A. Kosower, *All next-to-maximally helicity-violating one-loop gluon amplitudes in $N=4$ super-Yang-Mills theory*, hep-th/0412210.

[5] R. Britto, F. Cachazo and B. Feng, *Generalized unitarity and one-loop amplitudes in $N=4$ super-Yang-Mills*, hep-th/0412103.

[6] F. Cachazo, *Holomorphic anomaly of unitarity cuts and one-loop gauge theory amplitudes*, hep-th/0410077.

[7] R. Britto, F. Cachazo and B. Feng, *Computing one-loop amplitudes from the holomorphic anomaly of unitarity cuts*, *Phys. Rev.* **D 71** (2005) 025012 [hep-th/0410179].

[8] S.J. Bidder, N.E.J. Bjerrum-Bohr, L.J. Dixon and D.C. Dunbar, *$N=1$ supersymmetric one-loop amplitudes and the holomorphic anomaly of unitarity cuts*, *Phys. Lett.* **B 606** (2005) 189 [hep-th/0410296].

[9] S.J. Bidder, N.E.J. Bjerrum-Bohr, D.C. Dunbar and W.B. Perkins, *One-loop gluon scattering amplitudes in theories with $N < 4$ supersymmetries*, *Phys. Lett.* **B 612** (2005) 75 [hep-th/0502028].

[10] R. Britto, E. Buchbinder, F. Cachazo and B. Feng, *One-loop amplitudes of gluons in SQCD*, hep-ph/0503132.

[11] M.T. Grisaru, H.N. Pendleton and P. van Nieuwenhuizen, *Supergravity and the S matrix*, *Phys. Rev.* **D 15** (1977) 996;
M.T. Grisaru and H.N. Pendleton, *Some properties of scattering amplitudes in supersymmetric theories*, *Nucl. Phys.* **B 124** (1977) 81;
S.J. Parke and T.R. Taylor, *Perturbative QCD utilizing extended supersymmetry*, *Phys. Lett.* **B 157** (1985) 81, erratum *ibid.* **B174** (1986) 465.

[12] Z. Kunszt, *Combined use of the calkul method and $N = 1$ supersymmetry to calculate QCD six parton processes*, *Nucl. Phys.* **B 271** (1986) 333.

[13] S.J. Parke and T.R. Taylor, *An amplitude for N gluon scattering*, *Phys. Rev. Lett.* **56** (1986) 2459;
F.A. Berends and W.T. Giele, *Recursive calculations for processes with N gluons*, *Nucl. Phys.* **B 306** (1988) 759.

[14] V.P. Nair, *A current algebra for some gauge theory amplitudes*, *Phys. Lett.* **B 214** (1988) 215.

[15] F.A. Berends and W. Giele, *The six gluon process as an example of Weyl-Van der Waerden spinor calculus*, *Nucl. Phys.* **B 294** (1987) 700;
M.L. Mangano, S.J. Parke and Z. Xu, *Duality and multi-gluon scattering*, *Nucl. Phys.* **B 298** (1988) 653;
M.L. Mangano, *The color structure of gluon emission*, *Nucl. Phys.* **B 309** (1988) 461.

[16] M.L. Mangano and S.J. Parke, *Multiparton amplitudes in gauge theories*, *Phys. Rept.* **200** (1991) 301.

[17] Z. Xu, D.-H. Zhang and L. Chang, *Helicity amplitudes for multiple bremsstrahlung in massless nonabelian gauge theories*, *Nucl. Phys.* **B 291** (1987) 392.

[18] D.A. Kosower, *Light cone recurrence relations for QCD amplitudes*, *Nucl. Phys.* **B 335** (1990) 23.

[19] Z. Bern and D.A. Kosower, *Color decomposition of one loop amplitudes in gauge theories*, *Nucl. Phys.* **B 362** (1991) 389.

[20] Z. Bern, L.J. Dixon, D.C. Dunbar and D.A. Kosower, *One loop n point gauge theory amplitudes, unitarity and collinear limits*, *Nucl. Phys.* **B 425** (1994) 217 [hep-ph/9403226].

[21] Z. Bern and D.A. Kosower, *Efficient calculation of one loop QCD amplitudes*, *Phys. Rev. Lett.* **66** (1991) 1669; *The computation of loop amplitudes in gauge theories*, *Nucl. Phys.* **B 379** (1992) 451;
Z. Bern, *A compact representation of the one loop N gluon amplitude*, *Phys. Lett.* **B 296** (1992) 85;
Z. Bern, D.C. Dunbar and T. Shimada, *String based methods in perturbative gravity*, *Phys. Lett.* **B 312** (1993) 277 [hep-th/9307001];
Z. Bern and D.C. Dunbar, *A mapping between feynman and string motivated one loop rules in gauge theories*, *Nucl. Phys.* **B 379** (1992) 562;
D.C. Dunbar and P.S. Norridge, *Infinities within graviton scattering amplitudes*, *Class. and Quant. Grav.* **14** (1997) 351 [hep-th/9512084].

[22] Z. Bern, L.J. Dixon, D.C. Dunbar and D.A. Kosower, *Fusing gauge theory tree amplitudes into loop amplitudes*, *Nucl. Phys.* **B 435** (1995) 59 [hep-ph/9409265].

[23] Z. Bern, J.S. Rozowsky and B. Yan, *Two-loop four-gluon amplitudes in $N = 4$ super-Yang-Mills*, *Phys. Lett.* **B 401** (1997) 273 [hep-ph/9702424];
C. Anastasiou, Z. Bern, L.J. Dixon and D.A. Kosower, *Planar amplitudes in maximally supersymmetric Yang-Mills theory*, *Phys. Rev. Lett.* **91** (2003) 251602 [hep-th/0309040];
Z. Bern, L.J. Dixon and V.A. Smirnov, *Iteration of planar amplitudes in maximally supersymmetric Yang-Mills theory at three loops and beyond*, hep-th/0505205.

[24] W. Siegel, *Supergravity supergraphs*, *Phys. Lett.* **B 84** (1979) 197;
D.M. Capper, D.R.T. Jones and P. van Nieuwenhuizen, *Regularization by dimensional reduction of supersymmetric and nonsupersymmetric gauge theories*, *Nucl. Phys.* **B 167** (1980) 479;
L.V. Avdeev and A.A. Vladimirov, *Dimensional regularization and supersymmetry*, *Nucl. Phys.* **B 219** (1983) 262.

[25] Z. Kunszt, A. Signer and Z. Trócsányi, *One loop helicity amplitudes for all $2 \to 2$ processes in QCD and $N = 1$ supersymmetric Yang-Mills theory*, *Nucl. Phys.* **B 411** (1994) 397 [hep-ph/9305239].

[26] G. Georgiou and V.V. Khoze, *Tree amplitudes in gauge theory as scalar MHV diagrams*, *JHEP* **05** (2004) 070 [hep-th/0404072];
J.-B. Wu and C.-J. Zhu, *MHV vertices and fermionic scattering amplitudes in gauge theory with quarks and gluinos*, *JHEP* **09** (2004) 063 [hep-th/0406146];
X. Su and J.-B. Wu, *Six-quark amplitudes from fermionic MHV vertices*, *Mod. Phys. Lett.* **A 20** (2005) 1065 [hep-th/0409228].

[27] L.J. Dixon, E.W.N. Glover and V.V. Khoze, *MHV rules for Higgs plus multi-gluon amplitudes*, *JHEP* **12** (2004) 015 [hep-th/0411092];
Z. Bern, D. Forde, D.A. Kosower and P. Mastrolia, *Twistor-inspired construction of electroweak vector boson currents*, *Phys. Rev.* **D 72** (2005) 025006 [hep-ph/0412167];
S.D. Badger, E.W.N. Glover and V.V. Khoze, *MHV rules for Higgs plus multi-parton amplitudes*, *JHEP* **03** (2005) 023 [hep-th/0412275].

[28] R. Britto, F. Cachazo and B. Feng, *New recursion relations for tree amplitudes of gluons*, *Nucl. Phys.* **B 715** (2005) 499 [hep-th/0412308];
R. Britto, F. Cachazo, B. Feng and E. Witten, *Direct proof of tree-level recursion relation in Yang-Mills theory*, *Phys. Rev. Lett.* **94** (2005) 181602 [hep-th/0501052].

[29] R. Britto, B. Feng, R. Roiban, M. Spradlin and A. Volovich, *All split helicity tree-level gluon amplitudes*, *Phys. Rev.* **D 71** (2005) 105017 [hep-th/0503198].

[30] M.-x. Luo and C.-k. Wen, *Recursion relations for tree amplitudes in super gauge theories*, *JHEP* **03** (2005) 004 [hep-th/0501121].

[31] J. Bedford, A. Brandhuber, B. Spence and G. Travaglini, *A recursion relation for gravity amplitudes*, *Nucl. Phys.* **B 721** (2005) 98 [hep-th/0502146].

[32] F. Cachazo and P. Svrcek, *Tree level recursion relations in general relativity*, hep-th/0502160.

[33] N.E.J. Bjerrum-Bohr, D.C. Dunbar and H. Ita, *Six-point one-loop $N = 8$ supergravity nmhv amplitudes and their IR behaviour*, hep-th/0503102.

[34] R.K. Ellis and J.C. Sexton, *QCD radiative corrections to parton parton scattering*, *Nucl. Phys.* **B 269** (1986) 445;
M.B. Green, J.H. Schwarz and L. Brink, $N = 4$ *Yang-Mills and* $N = 8$ *supergravity as limits of string theories*, *Nucl. Phys.* **B 198** (1982) 474.

[35] Z. Bern, L.J. Dixon and D.A. Kosower, *One loop corrections to five gluon amplitudes*, *Phys. Rev. Lett.* **70** (1993) 2677 [hep-ph/9302280].

[36] Z. Bern, L.J. Dixon and D.A. Kosower, *One loop corrections to two quark three gluon amplitudes*, *Nucl. Phys.* **B 437** (1995) 259 [hep-ph/9409393].

[37] Z. Kunszt, A. Signer and Z. Trócsányi, *One loop radiative corrections to the helicity amplitudes of QCD processes involving four quarks and one gluon*, *Phys. Lett.* **B 336** (1994) 529 [hep-ph/9405386].

[38] E.W.N. Glover and D.J. Miller, *The one-loop QCD corrections for* $\gamma* \to q\bar{q}q\bar{q}$, *Phys. Lett.* **B 396** (1997) 257 [hep-ph/9609474].

[39] A. Brandhuber, B. Spence and G. Travaglini, *One-loop gauge theory amplitudes in* $N = 4$ *super Yang-Mills from MHV vertices*, *Nucl. Phys.* **B 706** (2005) 150 [hep-th/0407214].

[40] C. Quigley and M. Rozali, *One-loop mhv amplitudes in supersymmetric gauge theories*, *JHEP* **01** (2005) 053 [hep-th/0410278].

[41] J. Bedford, A. Brandhuber, B. Spence and G. Travaglini, *A twistor approach to one-loop amplitudes in* $N = 1$ *supersymmetric Yang-Mills theory*, *Nucl. Phys.* **B 706** (2005) 100 [hep-th/0410280].

[42] I. Bena, Z. Bern and D.A. Kosower, *Twistor-space recursive formulation of gauge theory amplitudes*, *Phys. Rev.* **D 71** (2005) 045008 [hep-th/0406133].

[43] R. Britto, F. Cachazo and B. Feng, *Coplanarity in twistor space of* $N = 4$ *next-to-MHV one-loop amplitude coefficients*, *Phys. Lett.* **B 611** (2005) 167 [hep-th/0411107].

[44] S.J. Bidder, N.E.J. Bjerrum-Bohr, D.C. Dunbar and W.B. Perkins, *Twistor space structure of the box coefficients of* $N = 1$ *one-loop amplitudes*, *Phys. Lett.* **B 608** (2005) 151 [hep-th/0412023].

[45] R.J. Eden, P.V. Landshoff, D.I. Olive, J.C. Polkinghorne, *The analytic S matrix*, Cambridge University Press, Cambridge 1966.

[46] Z. Bern, L.J. Dixon and D.A. Kosower, *One-loop amplitudes for* e^+e^- *to four partons*, *Nucl. Phys.* **B 513** (1998) 3 [hep-ph/9708239]; *A two-loop four-gluon helicity amplitude in QCD*, *JHEP* **01** (2000) 027 [hep-ph/0001001]; *Two-loop* $g \to gg$ *splitting amplitudes in QCD*, *JHEP* **08** (2004) 012 [hep-ph/0404293];
Z. Bern, L.J. Dixon, D.C. Dunbar and D.A. Kosower, *One-loop self-dual and* $N = 4$ *super Yang-Mills*, *Phys. Lett.* **B 394** (1997) 105 [hep-th/9611127].

[47] Z. Bern, L.J. Dixon and D.A. Kosower, *On-shell recurrence relations for one-loop QCD amplitudes*, *Phys. Rev.* **D 71** (2005) 105013 [hep-th/0501240].

[48] Z. Bern, L.J. Dixon and D.A. Kosower, *The last of the finite loop amplitudes in QCD*, hep-ph/0505055.

[49] S.J. Parke, *Gluon and massless gluino scattering using* $N = 2$ *supersymmetry*, FERMILAB-CONF-86-044-T, *Invited talk given at Int. Symp. on Particle and Nuclear Physics*, Beijing, China, Sep 2-7, 1985.

[50] R. Roiban, M. Spradlin and A. Volovich, *Dissolving $N = 4$ loop amplitudes into QCD tree amplitudes*, *Phys. Rev. Lett.* **94** (2005) 102002 [hep-th/0412265].

[51] J.F. Gunion and Z. Kunszt, *Six quark subprocesses in QCD*, *Phys. Lett.* **B 176** (1986) 163.

[52] Z. Xu, D.-H. Zhang and L. Chang, *Helicity amplitudes for multiple bremsstrahlung in massless nonabelian gauge theories*, *Nucl. Phys.* **B 291** (1987) 392.

[53] M.-x. Luo and C.-k. Wen, *Compact formulas for all tree amplitudes of six partons*, *Phys. Rev.* **D 71** (2005) 091501 [hep-th/0502009].

[54] Z. Bern, N.E.J. Bjerrum-Bohr and D.C. Dunbar, *Inherited twistor-space structure of gravity loop amplitudes*, *JHEP* **05** (2005) 056 [hep-th/0501137].

[55] Z. Bern, L.J. Dixon, D.C. Dunbar, M. Perelstein and J.S. Rozowsky, *On the relationship between Yang-Mills theory and gravity and its implication for ultraviolet divergences*, *Nucl. Phys.* **B 530** (1998) 401 [hep-th/9802162]; *Perturbative relations between gravity and gauge theory*, *Class. and Quant. Grav.* **17** (2000) 979 [hep-th/9911194].